Good morning

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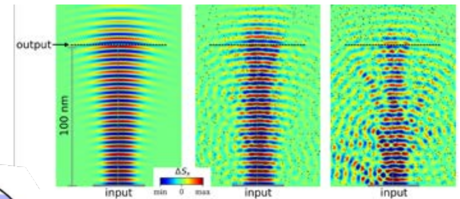
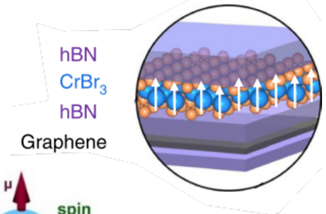
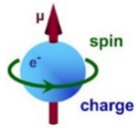
Nano Center of Excellence
http://cmt.uantwerpen.be

From nanophysics to functional materials



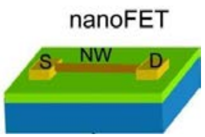


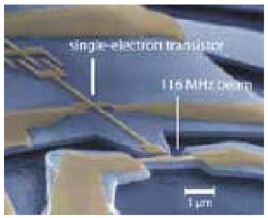


Spintronics & magnonics

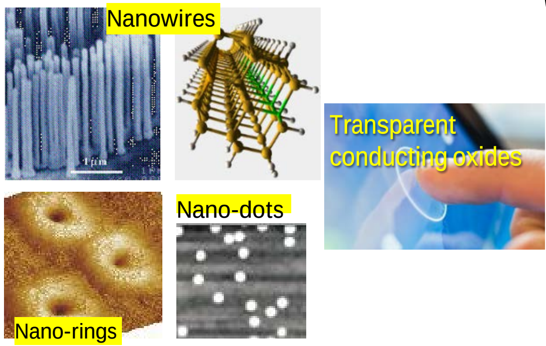
Semiconductors

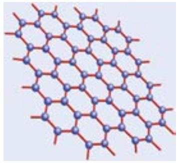
Nano-electronics



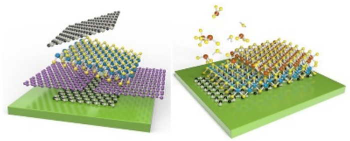


Nano-photonics





2D materials:
graphene, hBN, TMD,
vdW heterostructures



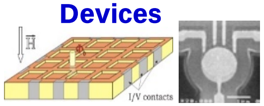
High performance computing

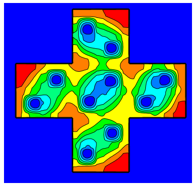
Classical: Monte Carlo, Molecular Dynamics
Quantum: DFT, AIMD, multicomponent, mean-field
Num. methods: parallel, high-throughput, AI, ML
Infrastructure: CalcUA, VSC, LUMI

Superconductors

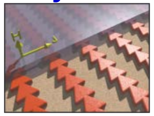
Materials & mechanisms

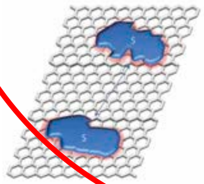
Devices



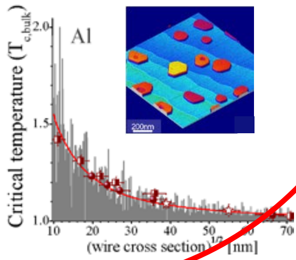


Hybrids



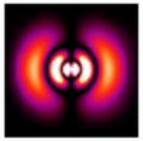


Nanoscale



Classical systems

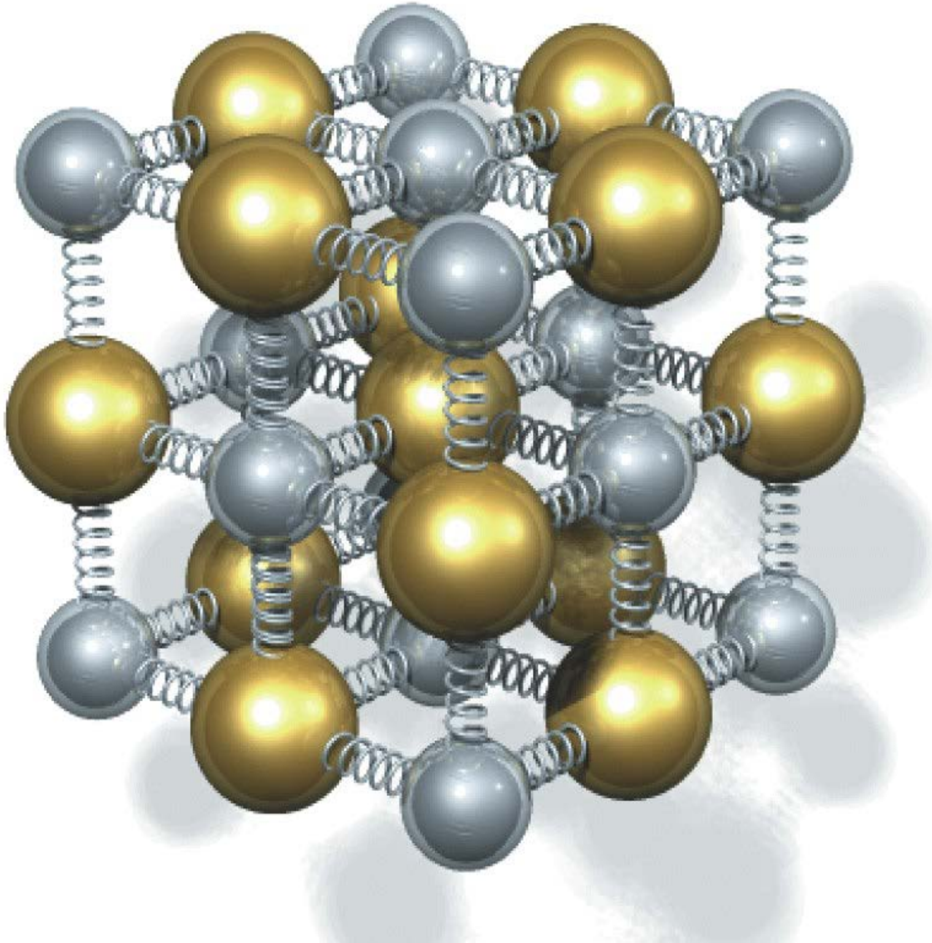
e.g. colloids, (bi)metallic i magnetic
na
ap
wi
pr



pybinding TBStudio GLACE MuMax^4 KITE

Trivia

- Everyone knows a metal can conduct electricity, i.e. electrons can flow through the metal when pushed by electric field (say, a battery).
- Of course, we also know that the wire is made of atoms.



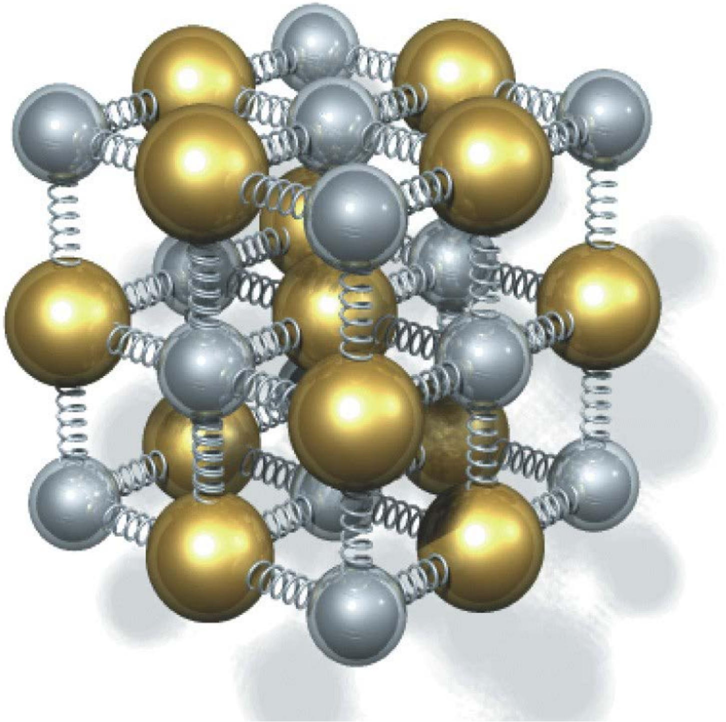
Resistance question

Suppose we have a perfect crystal of a metal in which we produce an electric current. The electrons in the metal:

A. Collide with the atoms, causing electrical resistance;

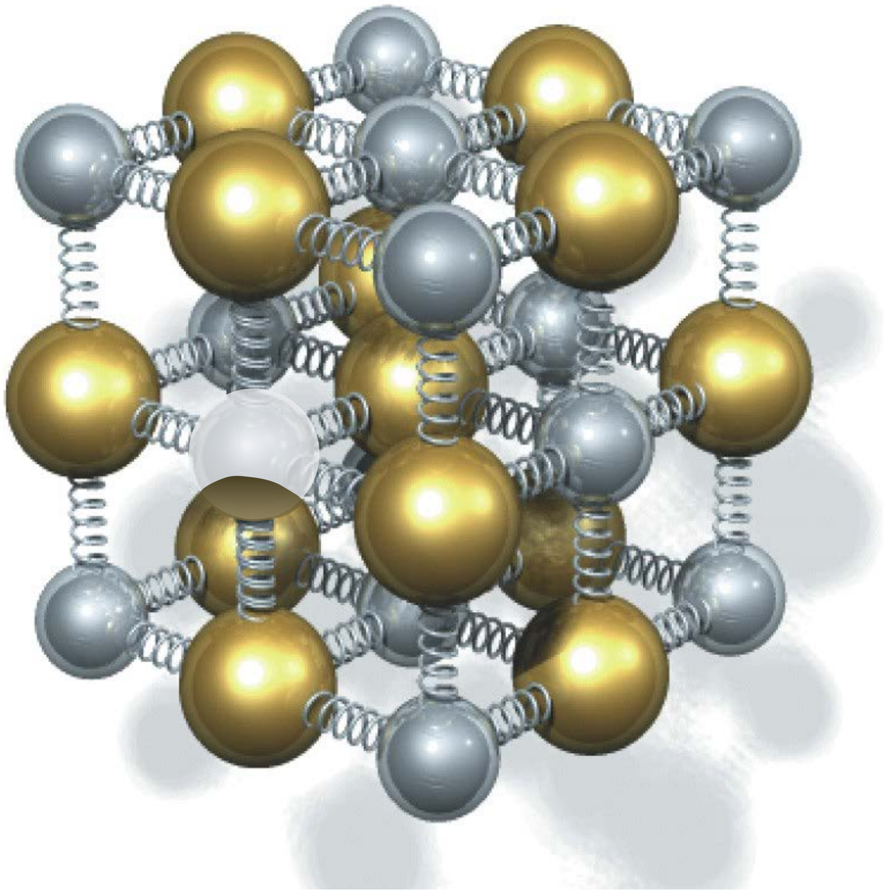
B. Twist between atoms, causing electrical resistance;

C. Propagate through the crystal without any electrical resistance?



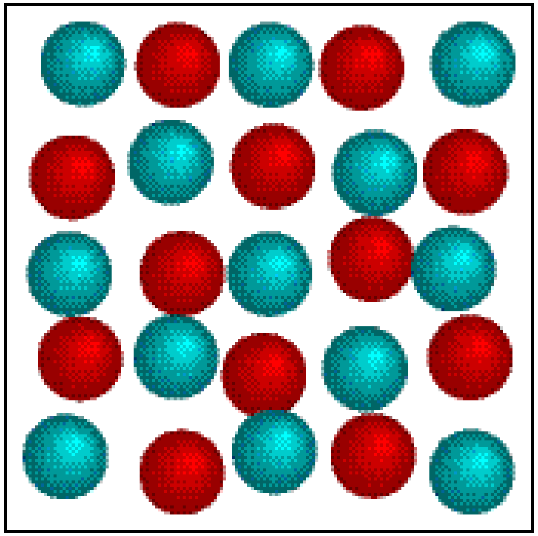
If all atoms are perfectly in place, the electron moves through the wire without any resistance!

The real world



Some missing atoms (defects)

Vibrating atoms!



Electron scatters -> resistance

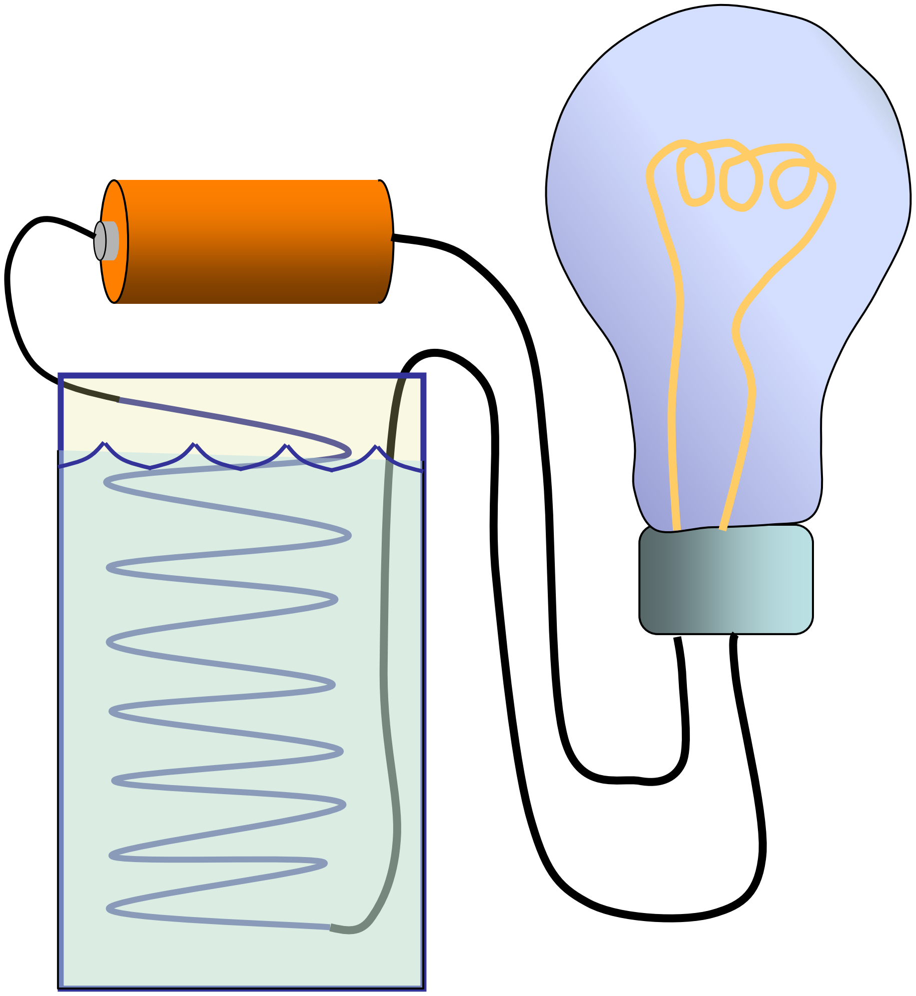
Temperature-dependent resistance

Suppose we cool down the wire that carries the electrical current to the light bulb. The light will:

A. Get brighter;

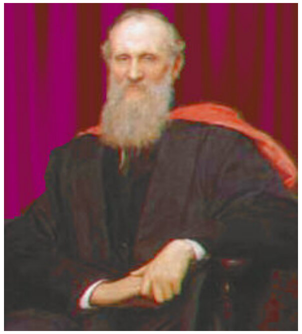
B. Get dimmer;

C. Stay same?



As temperature is decreased, the interaction of electrons with vibrating atoms and impurities decreases - resistance decreases - more current flows through the wire, and light bulb gets brighter.

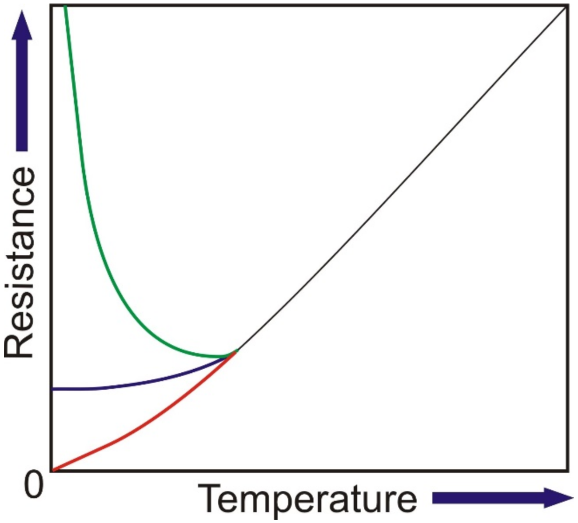
What happens at the lowest temperature?



Kelvin (1824-1907):
Electrons freeze and resistance increases as T decreases

Drude (1863-1906):
Electrons bounce off vibrating atoms and impurities

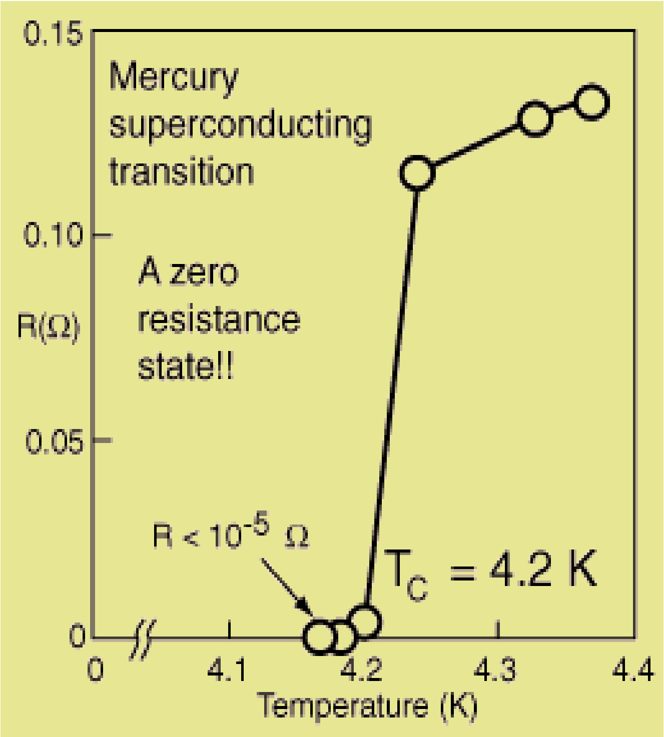
Onnes (1853-1926):
Resistance continues to decrease, finally reaching zero at zero temperature



Surprise

Heike Kamerlingh Onnes

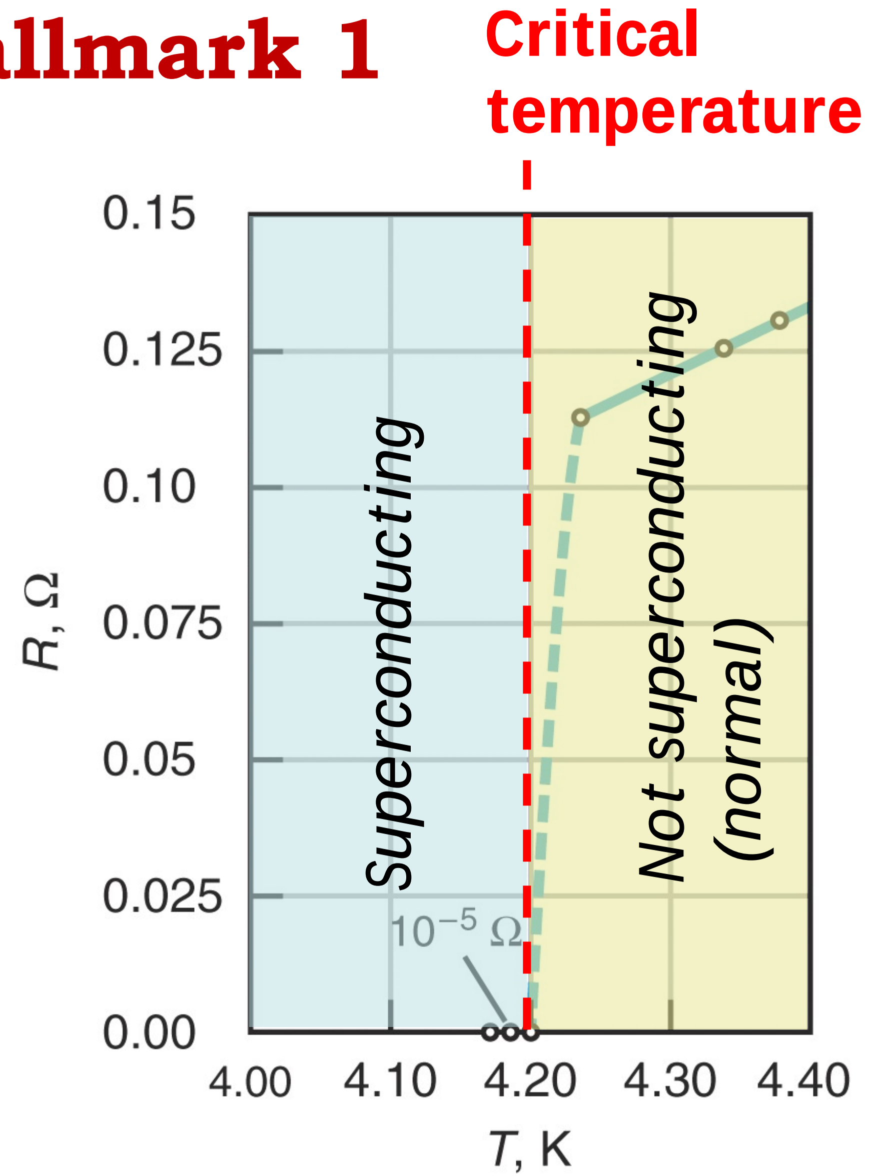
- 1908 - liquefied helium (~4 K = - 269°C)
- Annealed pure mercury (Hg)
- 1911- investigated low temperature resistance of mercury
- Found resistance dropped abruptly to zero at 4.2 K!!!
- 1913 - Nobel Prize in physics



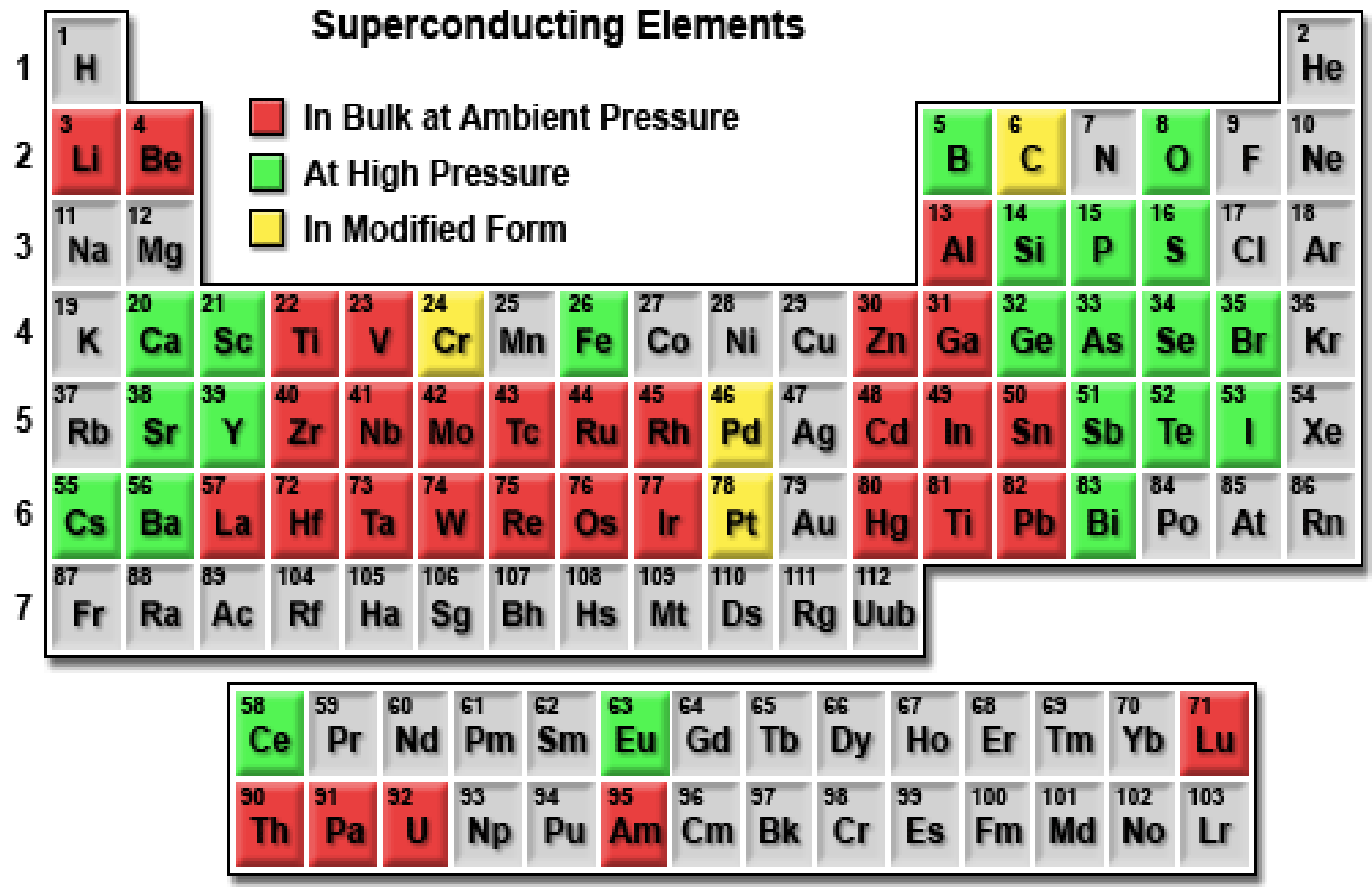
Superconductivity: hallmark 1

- Superconductors are materials that have exactly zero electrical resistance.
- Superconductivity occurs at temperatures below a critical temperature, T_c.
- In most cases this temperature is far below room temperature.

Element	Critical T (K)	(˚C)
Aluminum	1.75	-271
Mercury	4.15	-269
Lead	7.2	-266
Tin	3.72	-269
Niobium	9.25	-264

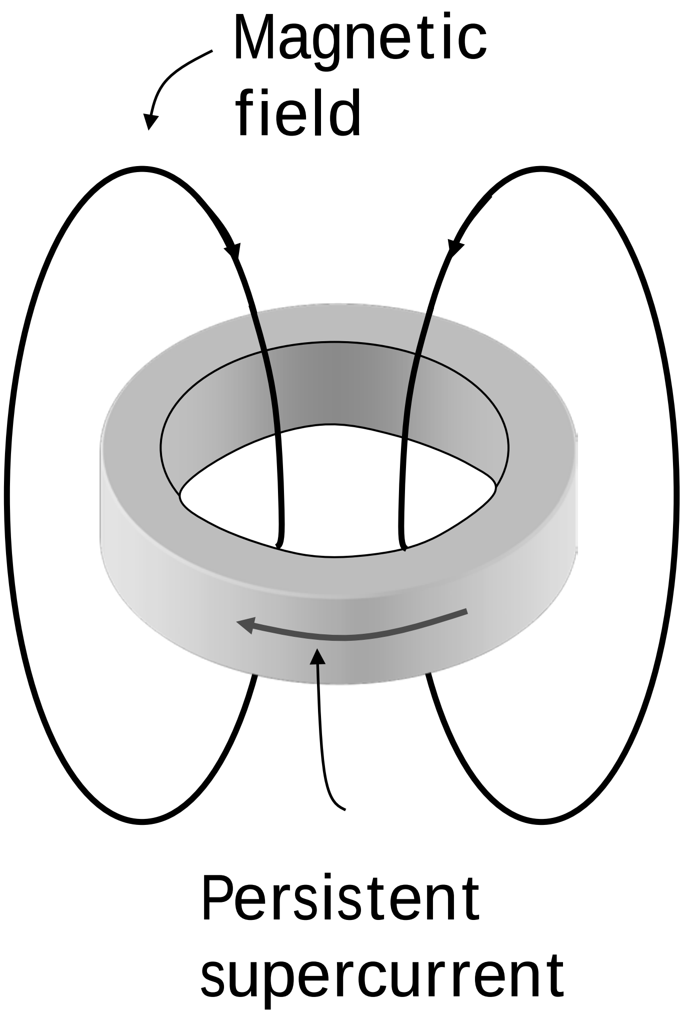


Superconducting elements



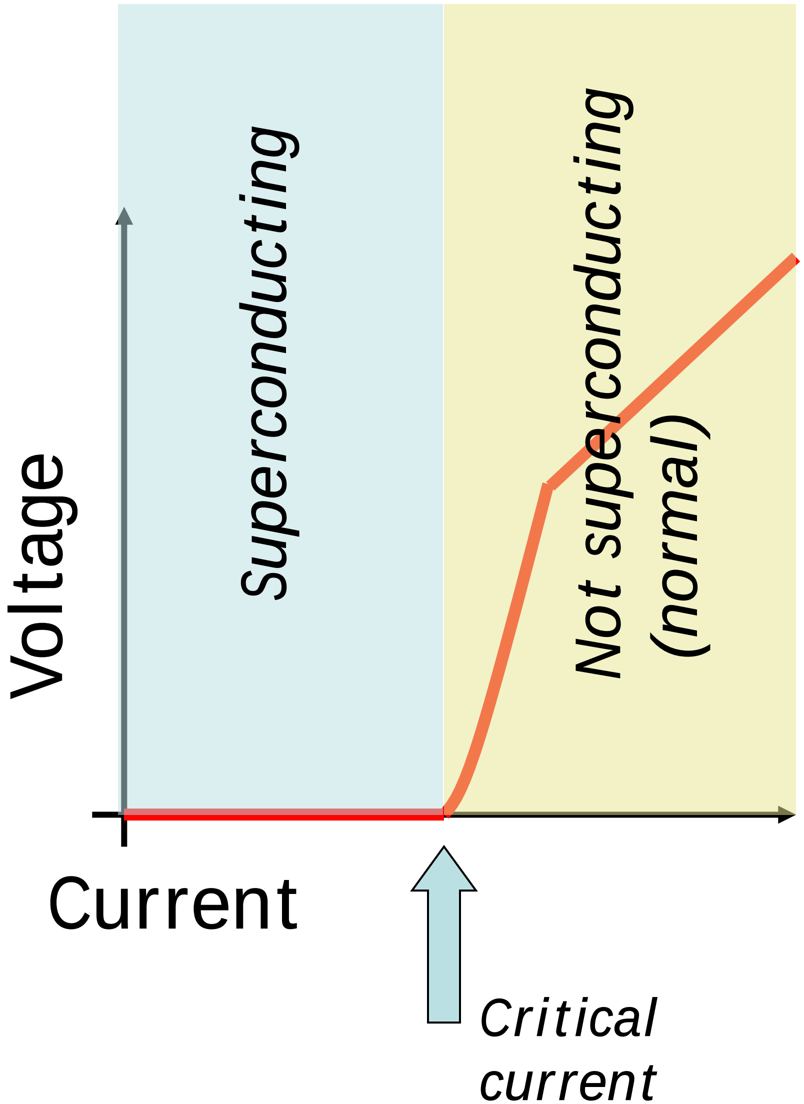
Persistent currents

- How zero is zero?
- EXACTLY!
- One of the first experiments was to set up a persistent current in a Pb ring (Holst *et al.* 1913).
- The magnitude of the current was monitored via the generated magnetic field.
- No current decay detected (until the WWI)



Critical current

- If the current is too large, superconductivity is destroyed.
- Maximum current for zero resistance is called the **critical current**.
- For larger currents, the voltage is no longer zero, and power is dissipated.

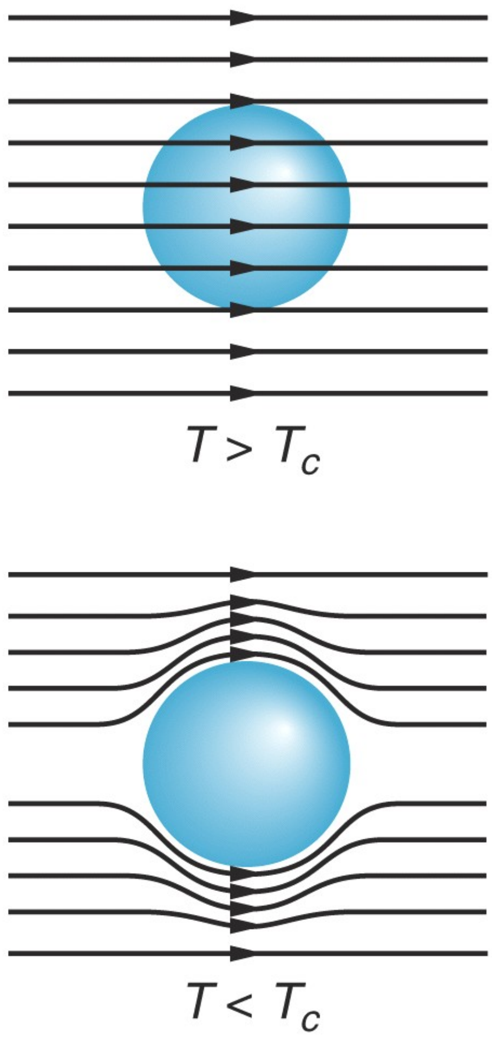


Superconductivity: hallmark 2

The Meissner effect

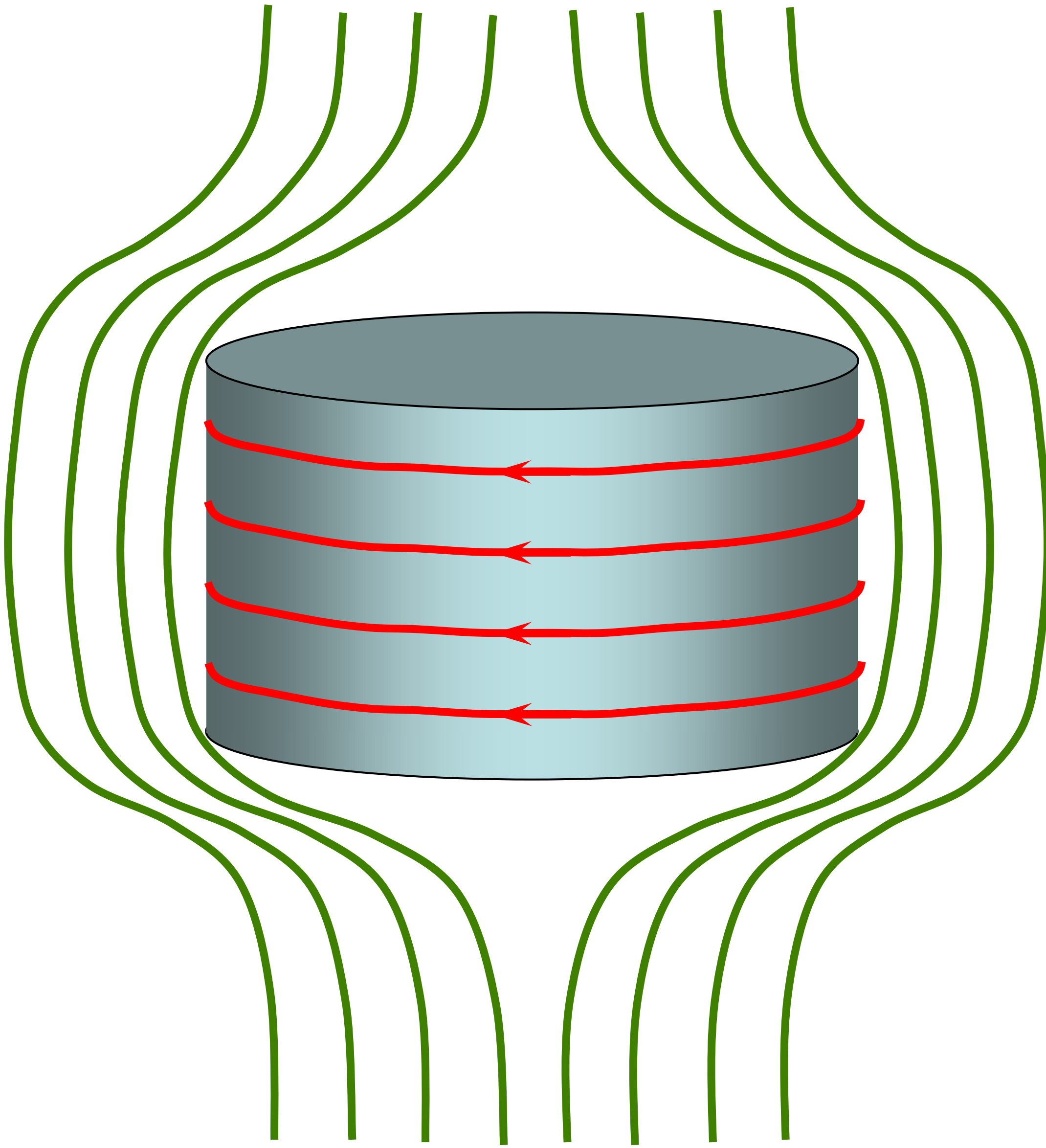
Meissner and Ochsenfeld in 1933:

- Diamagnetic response to a magnetic field.
- For small magnetic fields a superconductor will spontaneously expel all magnetic flux.
- Above the critical temperature, this effect is not observed.



Meissner effect

- Apply uniform magnetic field.
- Superconductor responds with circulating current.
- Produces own magnetic field.
- Finally, field is zero inside the superconductor, enhanced outside.



Question

A superconductor expels an applied magnetic field with a circulating supercurrent that generates a cancelling magnetic field.

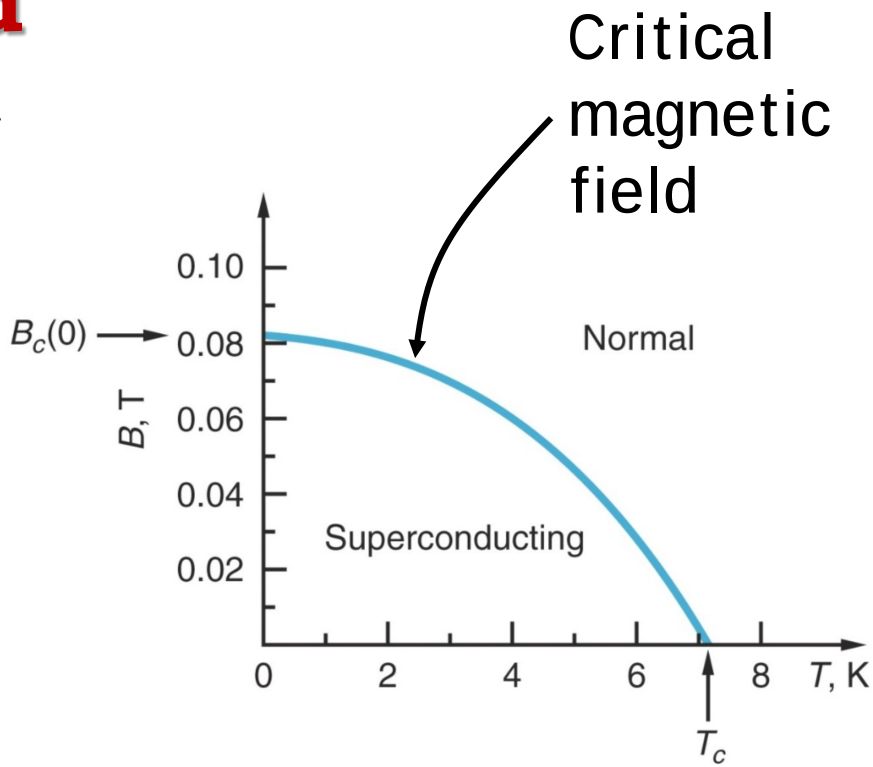
A superconductor has a maximum supercurrent it can carry before losing superconductivity.

When the applied magnetic field is increased to larger and larger values, the superconductor:

A. Continues to expel the field;
B. Expels only part of the field;
C. Loses superconductivity?

Critical magnetic field

- Magnetic field is screened out by the screening current.
- Larger fields require larger screening currents.
- Screening currents cannot be larger than the critical current.
- Thus there exists a maximal magnetic field which can be screened.
- Above this field, superconductivity is destroyed.



Superconductor phase diagram

(Type I)

Critical magnetic field

- It was one of Onnes' disappointments that even small magnetic fields destroyed superconductivity.
- Superconductivity seemed a fragile effect
  - Only observed at low temperature;
  - Destroyed by small magnetic fields.

Theory of superconductivity

Einstein, Bohr,
Dirac, Bloch,
Landau…

all agonizing

London equations

In 1934, brothers London derived their first equation from Newton's law:

$$E = \frac{d}{dt}\left(\Lambda J_s\right) \qquad \Lambda = \frac{m}{n_s e^2}$$

London equations

- Newton's law (inertial response) for applied electric field

$$F = m\frac{d}{dt}(v_s) \quad \Rightarrow \quad eE = m\frac{d}{dt}\left(\frac{J_S}{n_s e}\right) \quad \Rightarrow \quad \boxed{E = \frac{d}{dt}(\Lambda J_S) \qquad \Lambda = \frac{m}{n_s e^2}}$$

Supercurrent density is

$$J_s = n_s e v_s$$

Faraday's law

$$\frac{n_s e^2 E}{m} = \frac{dJ_S}{dt} \quad \Rightarrow \quad \vec{\nabla}\times\frac{n_s e^2 \vec{E}}{m} = \vec{\nabla}\times\frac{d\vec{J}_S}{dt} \quad \Rightarrow \quad -\frac{n_s e^2}{m}\frac{d\vec{B}}{dt} = \vec{\nabla}\times\frac{d\vec{J}_S}{dt}$$

$$\Rightarrow \quad \frac{d}{dt}\left[\vec{\nabla}\times\vec{J}_S + \frac{n_s e^2}{m}\vec{B}\right] = 0 \quad \Rightarrow \quad \boxed{\vec{\nabla}\times\vec{J}_S = -\frac{n_s e^2}{m}\vec{B}}$$

We know B = 0 inside superconductors

London equations

London equations

$$E = \frac{d}{dt}(\Lambda J_s) \qquad \Lambda = \frac{m}{n_s e^2}$$

$$\vec{\nabla} \times \vec{J}_s = -\frac{n_s e^2}{m}\vec{B}$$

Ampere's law

$$\vec{\nabla} \times \vec{B} = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu_0 \vec{\nabla} \times J$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \vec{\nabla}^2 \vec{B} = -\mu_0 \frac{n_s e^2}{m}\vec{B}$$

=0; Gauss's law for electrostatics

$$\vec{\nabla}^2 \vec{B} = \mu_0 \frac{n_s e^2}{m}\vec{B}$$

London equations

In 1934, brothers London derived their first equation from Newton's law:

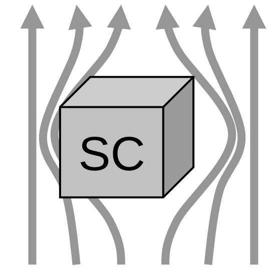
$$E = \frac{d}{dt}(\Lambda J_s) \qquad \Lambda = \frac{m}{n_s e^2}$$

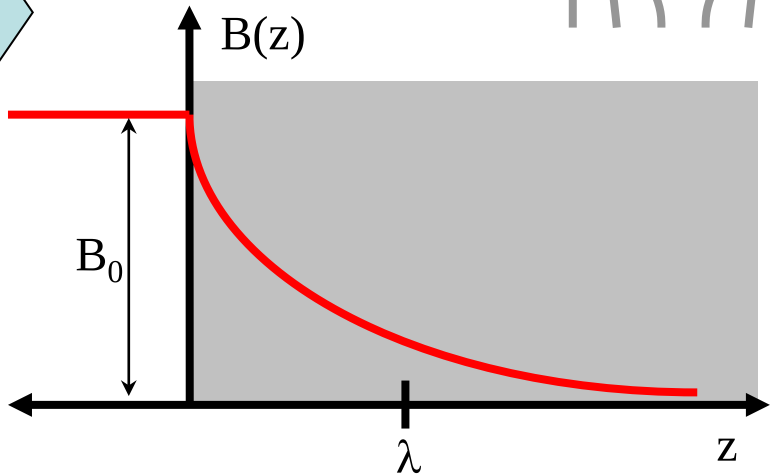
In 1935, using Faraday's law and the fact that B=0 inside the superconductor, they obtained:

$$\vec{\nabla} \times \vec{J}_S = -\frac{n_s e^2}{m}\vec{B}$$

$$\lambda^2 = \frac{m}{\mu_0 n_s e^2} \qquad \vec{\nabla}^2 \vec{B} = \frac{1}{\lambda^2}\vec{B}$$

$$B(z) = B_0 e^{-z/\lambda}$$

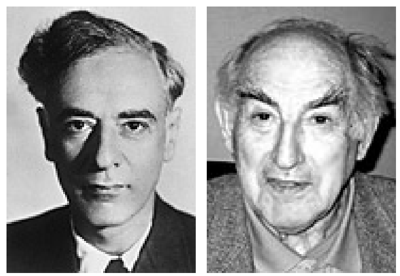




Magnetic (London) penetration depth

Theory of superconductivity

1950: Landau-Ginzburg theory



1911: superconductivity discovered

2024

1935: London theory

Ginzburg-Landau theory

- First consider zero magnetic field
- Order parameter ψ
- Associate with carrier density:

$$n_s = |\psi|^2$$

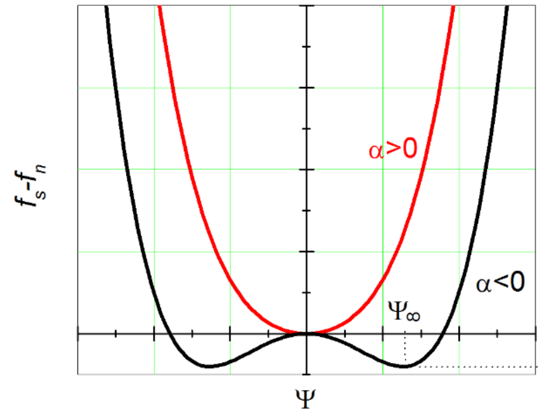
- Expand f in powers of $|\psi|^2$

→ To make sense, $\beta > 0$, $\alpha = \alpha(T)$

Free energy of the superconducting state

Free energy of the normal state

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4$$



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Ginzburg-Landau theory

- Momentum term:

$$H = \frac{p^2}{2m} + V, \quad p \to -i\hbar\nabla$$

- Now – include energy of the magnetic field

$$f_{magnetic} = +\frac{|B|^2}{2\mu_0}$$

- Classically, know that to include magnetic fields

$$p \to \left(-i\hbar\nabla - qA\right)$$

$$f_s - f_n = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 \Rightarrow$$

$$f_s - f_n = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\nabla - 2eA\right)\psi\right|^2 + \frac{|B|^2}{2\mu_0}$$

$$g = f - \mu_0 H \cdot M$$

The Ginzburg-Landau equations

Normal metallic system:

$$\frac{1}{2m}\left(-i\hbar\vec{\nabla}-\frac{e}{c}\vec{A}\right)^2\Psi+U\Psi=E\Psi$$

Schrödinger equation

Superconducting system:

$$\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}-\frac{e^*}{c}\vec{A}\right)^2\psi+\beta|\psi|^2\psi=-\alpha\psi$$

GL-I

$$\vec{j}_s=\frac{c}{4\pi}\vec{\nabla}\times\vec{h}=-\frac{i\hbar e^*}{2m^*}\left(\psi^*\vec{\nabla}\psi-\psi\vec{\nabla}\psi^*\right)-\frac{4e^2}{m^*c}|\psi|^2\vec{A}$$

GL-II

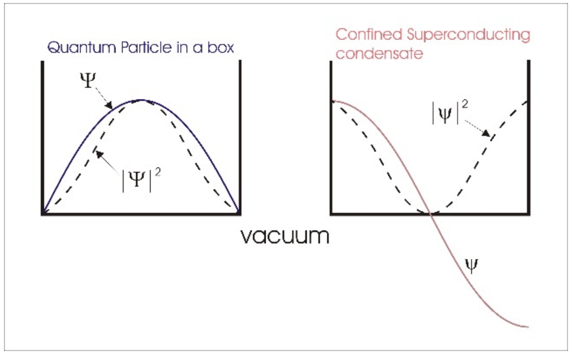
The boundary conditions:

$$\Psi\Psi^*\Big|_{boundary}=0$$

Normal metal – vacuum
Dirichlet BC

$$\left.\left(-i\hbar\vec{\nabla}-\frac{e^*}{c}\vec{A}\right)\right|_{\perp,boundary}\psi=0$$

SC – vacuum
Neumann BC



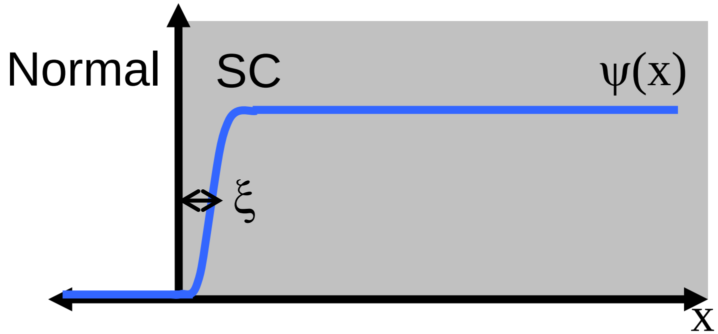
The coherence length

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2eA)^2\psi = 0$$

Take ψ real, normalize $\psi_\infty^{\ 2} = \frac{-\alpha}{\beta}$ → $$\alpha\left(\frac{\psi}{\psi_\infty}\right) - \alpha\left(\frac{\psi}{\psi_\infty}\right)^3 - \frac{\hbar^2}{2m}\nabla^2\left(\frac{\psi}{\psi_\infty}\right) = 0$$

Define $\Psi \equiv \frac{\psi}{\psi_\infty}$ → $$\frac{\hbar^2}{|\alpha(T)|2m}\nabla^2\Psi + \Psi - \Psi^3 = 0$$

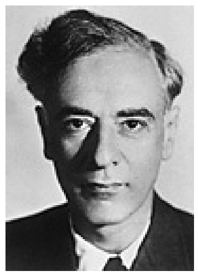
Linearize in ψ $\xi(T) = \sqrt{\frac{\hbar^2}{|\alpha(T)|2m}}$ → $$\nabla^2\Psi - \frac{2}{\xi^2(T)}\Psi = 0$$

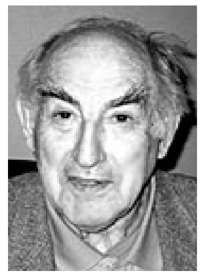


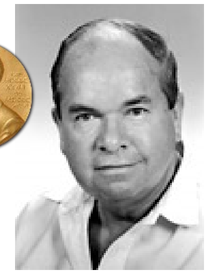
Theory of superconductivity

1950: Landau-Ginzburg theory

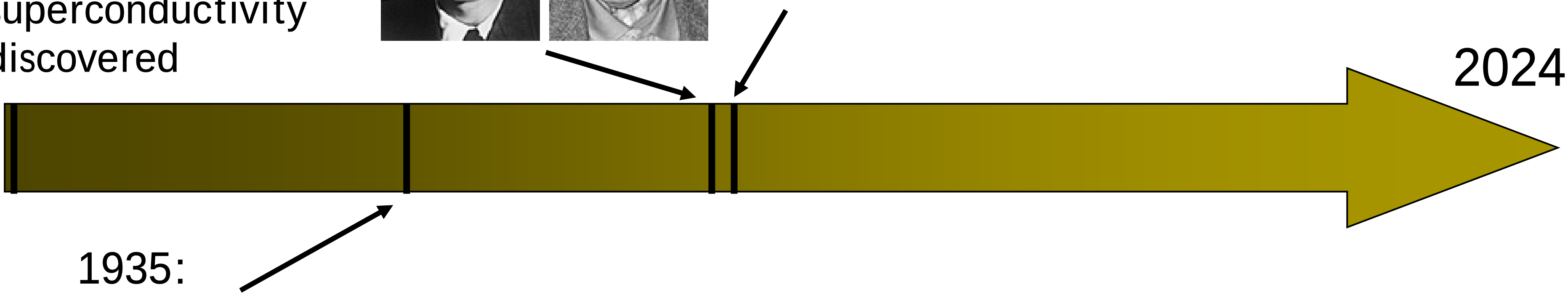
1953: Vortices (type-I/II superconductors)





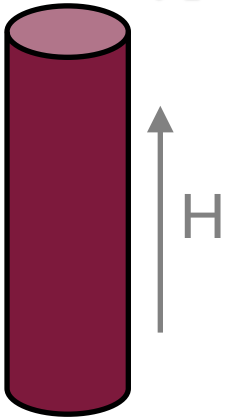


1911: superconductivity discovered

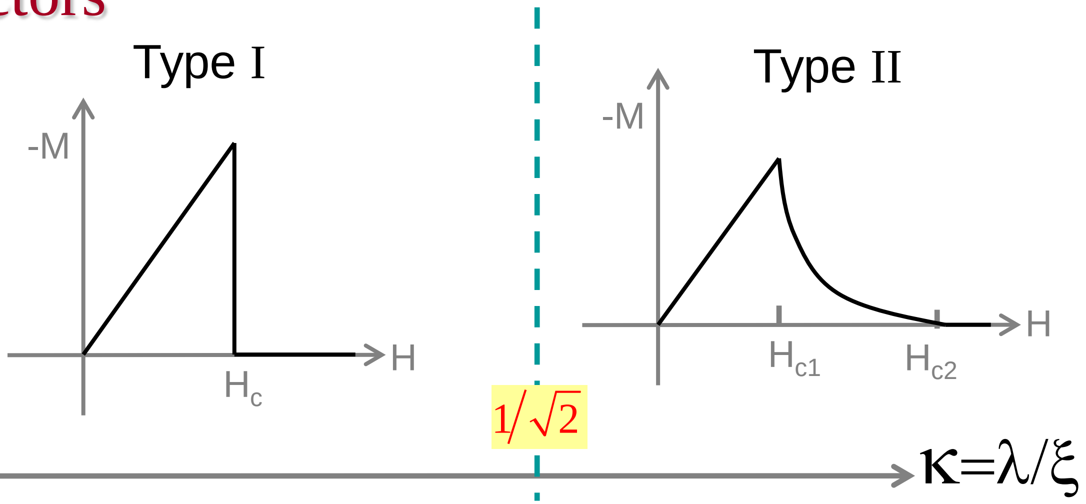


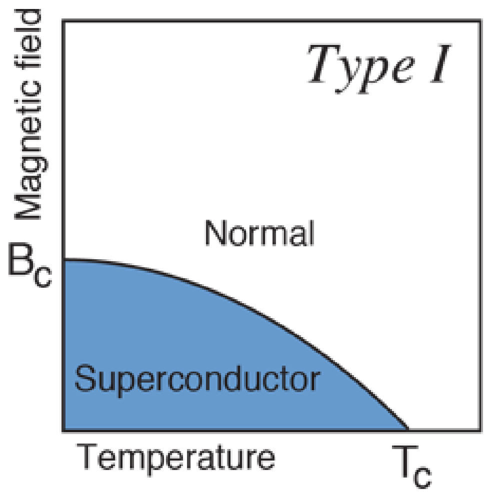
1935: London theory

Type-I vs. type-II superconductors

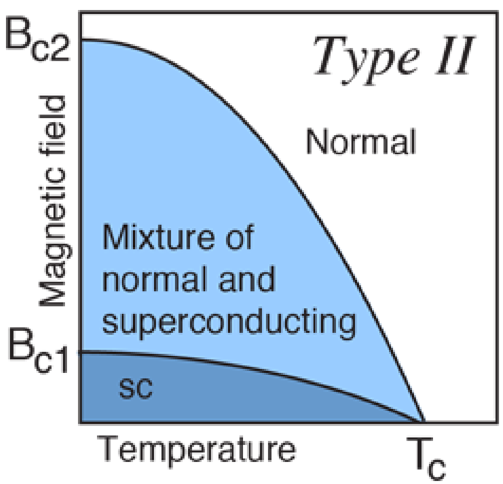


'Bulk' superconductors

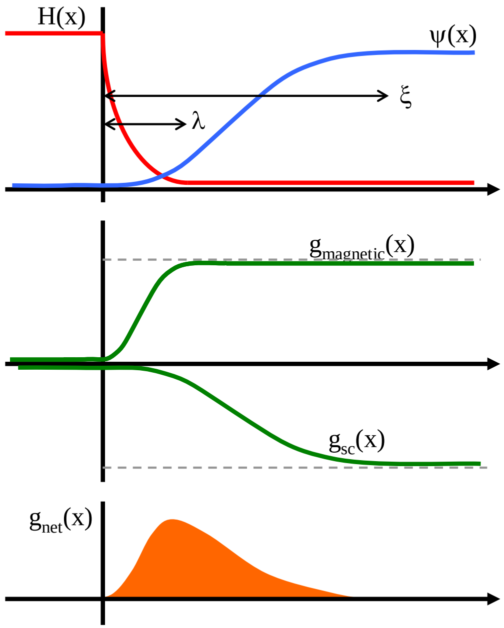




The difference is the energy of the S/N interface!



Type I



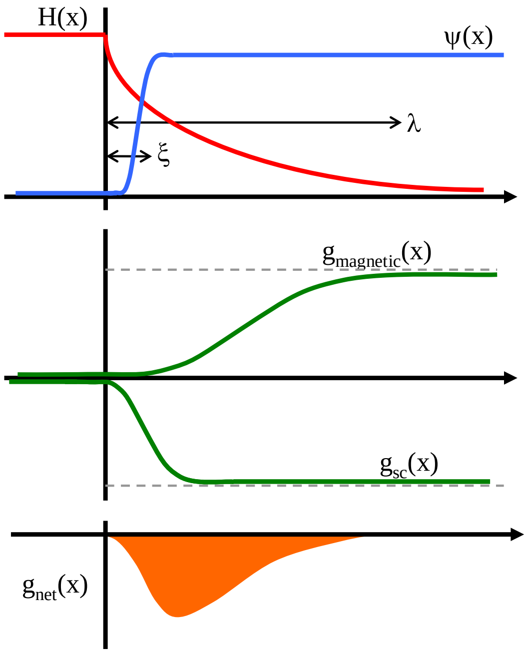
$$\kappa \equiv \frac{\lambda}{\xi} = \frac{1}{\sqrt{2}}$$

$\lambda << \xi$

- elemental superconductors

	ξ (nm)	λ (nm)	T_c (K)	H_{c2} (T)
Al	1600	50	1.2	.01
Pb	83	39	7.2	.08
Sn	230	51	3.7	.03

Type II

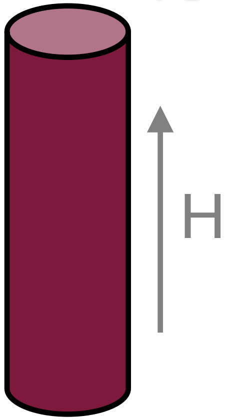


$\lambda >> \xi$

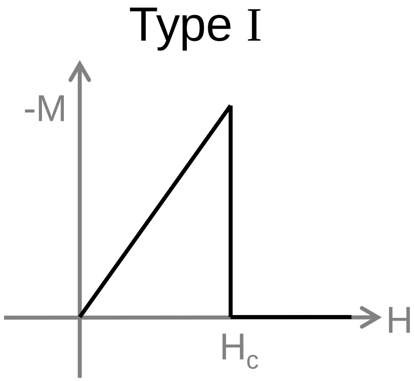
- predicted in 1950s by Abrikosov

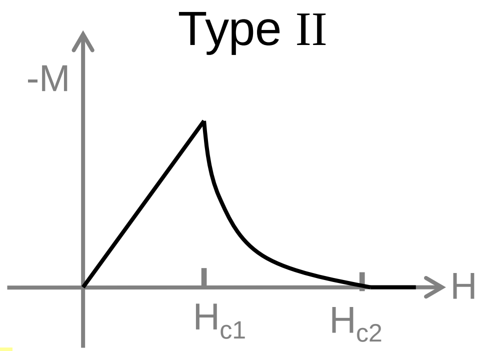
	ξ (nm)	λ (nm)	T_c (K)	H_{c2} (T)
Nb_3Sn	11	200	18	25
YBCO	1.5	200	92	150
MgB_2	5	185	37	14

Type-I vs. type-II superconductors



'Bulk' superconductors

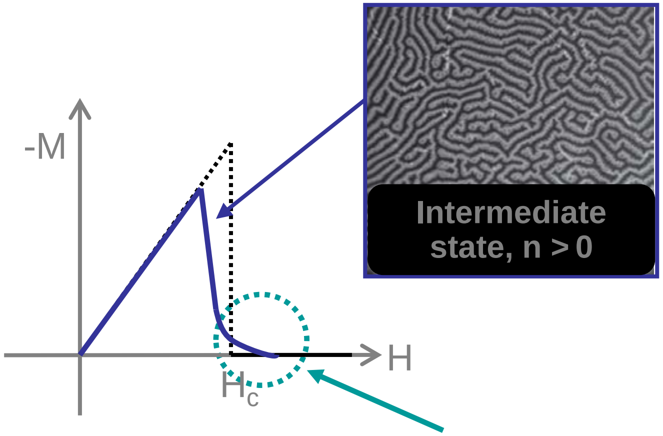




$1/\sqrt{2}$

$\kappa = \lambda/\xi$

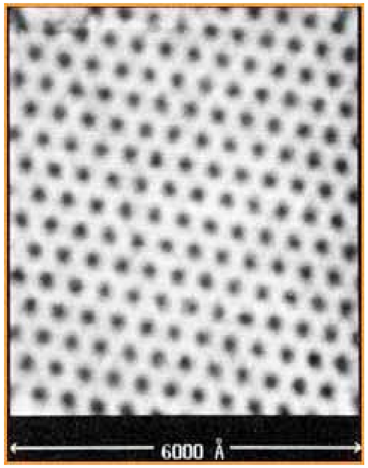
Intermediate state of type I superconductors



Surface superconductivity up to H_{c3}

The difference is the energy of the S/N interface!

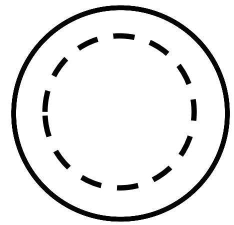
Intermediate state of type II superconductors



Hallmark 3: Flux quantization

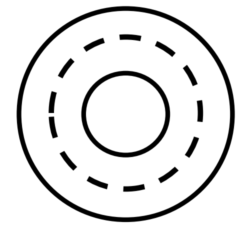
ψ needs to single-valued which forces a phase constraint:

Singly-connected



$$\oint \vec{\nabla}\theta \cdot \overrightarrow{d\ell} = 0$$

Multiply-connected



$$\oint \vec{\nabla}\theta \cdot \overrightarrow{d\ell} = 2\pi n$$

$$\frac{\hbar c}{e^*}\vec{\nabla}\theta = \vec{A} + \frac{m^* c}{n_s (e^*)^2}\vec{J}_s$$

Fluxoid quantized in units of

$\Phi_0 = \frac{hc}{2e}$ “flux quantum”

Fluxoid

$$\Phi' = \frac{\Phi_0}{2\pi}\oint \vec{\nabla}\theta \cdot \overrightarrow{d\ell} = n\,\Phi_0$$

$$\frac{\hbar c}{e^*}\oint \vec{\nabla}\theta \cdot \overrightarrow{d\ell} = \oint \vec{A} \cdot \overrightarrow{d\ell} + \frac{m^* c}{n_s (e^*)^2}\oint \vec{J}_s \cdot \overrightarrow{d\ell}$$

$$\Phi_0 = 2.07 \times 10^{-15}\,Wb$$
$$= 2.07 \times 10^{-7}\; G - cm^2$$

$$\frac{\hbar c}{e^*}(2\pi n) = \Phi + \frac{m^* c}{n_s^* (e^*)^2}\oint \vec{J}_s \cdot \overrightarrow{d\ell}$$

“fluxoid” — magnetic flux — kinetic flux

Flux quantization

VOLUME 7, NUMBER 2 PHYSICAL REVIEW LETTERS JULY 15, 1961

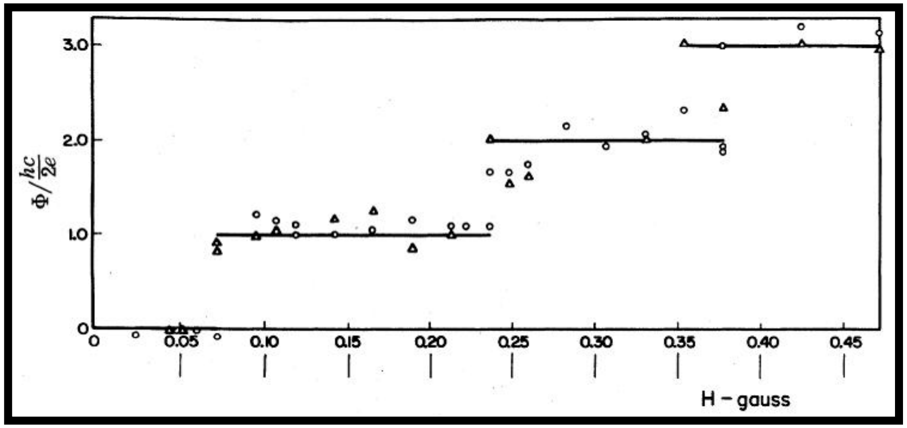
EXPERIMENTAL EVIDENCE FOR QUANTIZED FLUX IN SUPERCONDUCTING CYLINDERS*

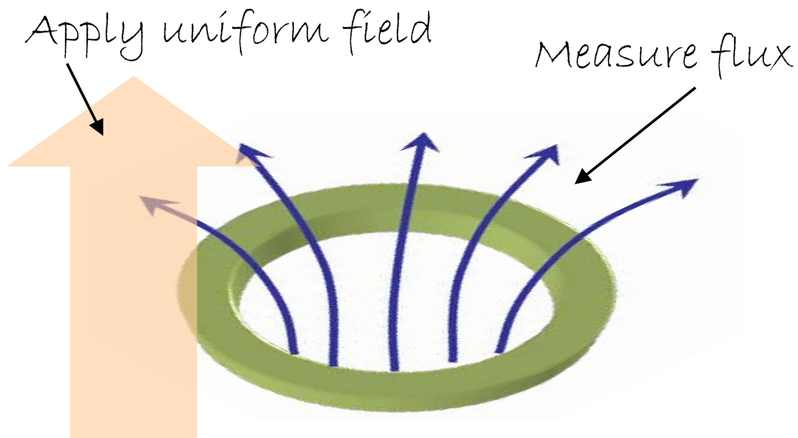
Bascom S. Deaver, Jr., and William M. Fairbank
Department of Physics, Stanford University, Stanford, California
(Received June 16, 1961)

We have observed experimentally quantized values of magnetic flux trapped in hollow superconducting cylinders. That such an effect might occur was originally suggested by London[1] and Onsager,[2] the predicted unit being hc/e. The quantized unit we find experimentally is not hc/e, but $hc/2e$ within experimental error.[3]

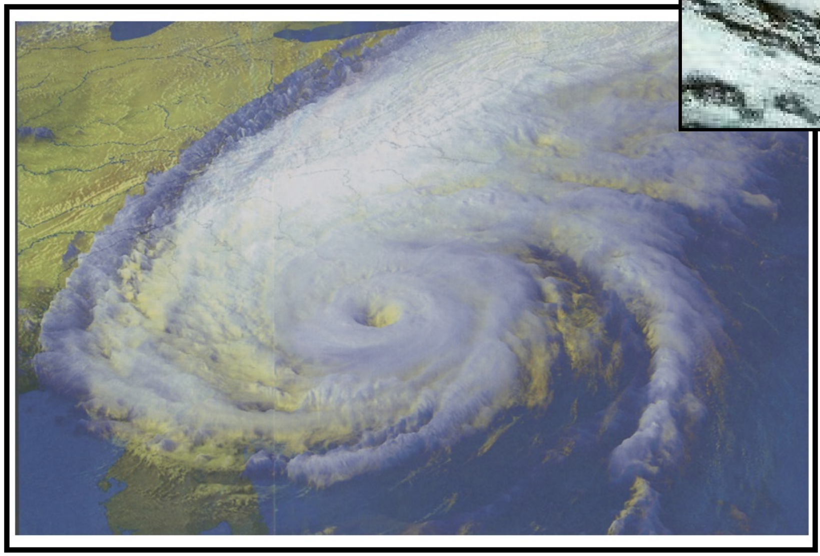
solute value of the flux for a given signal is calculated.

The diameter of each cylinder was measured with a microscope equipped with a micrometer eyepiece. X-ray photographs verified the dimensions of the tin cylinder after the application of the copper jacket. For the purpose of cal-

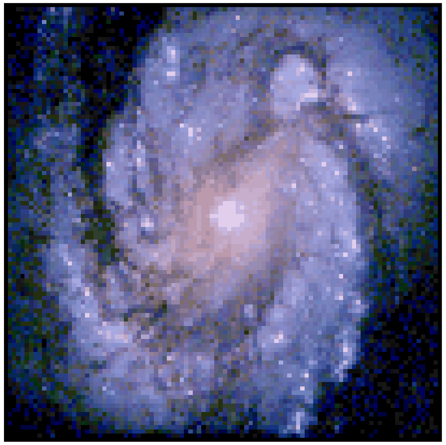




Vortices in nature



Great red spot on Jupiter and cloud vortex over Earth

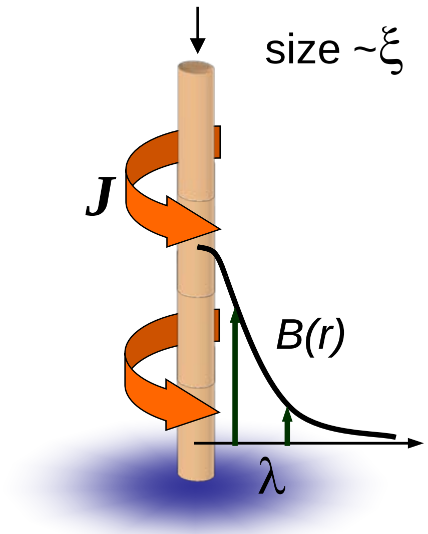


Spiral galaxy M-100

Tornado

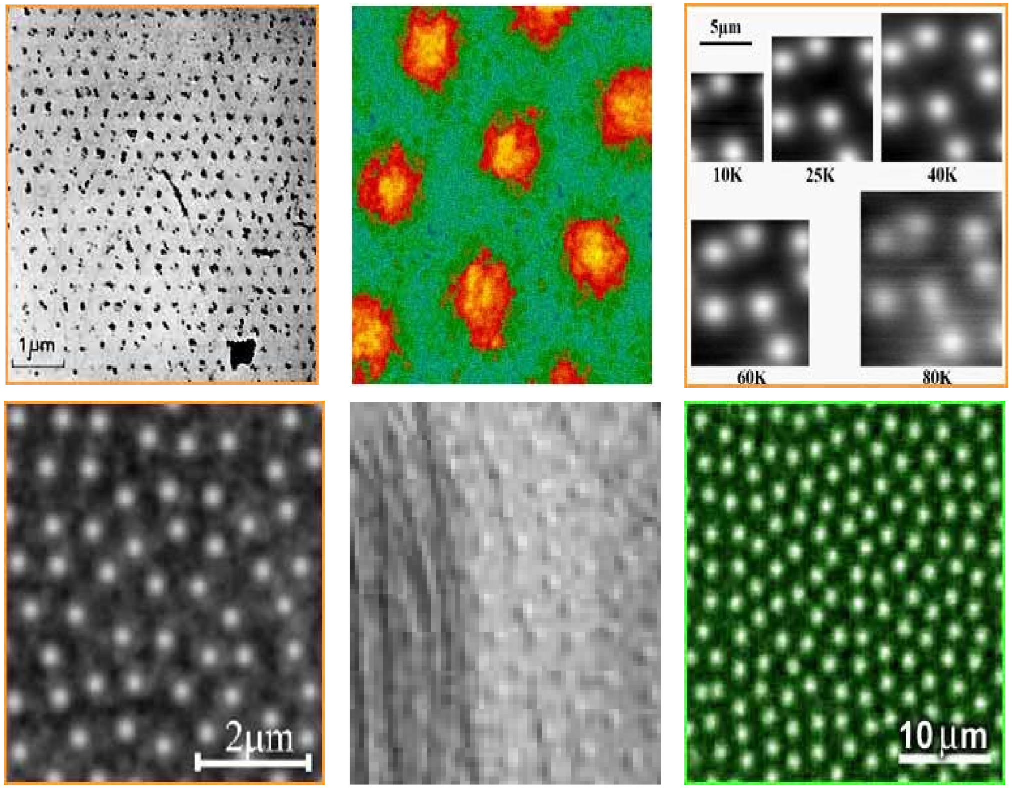
Superconductivity: hallmark 3

Flux quantization

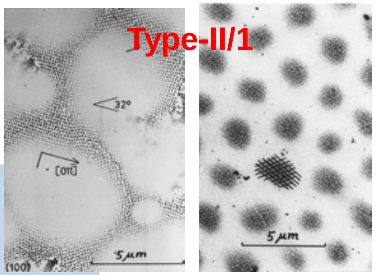


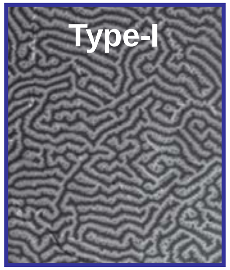
Carries **exactly 1 quantum** of magnetic flux.

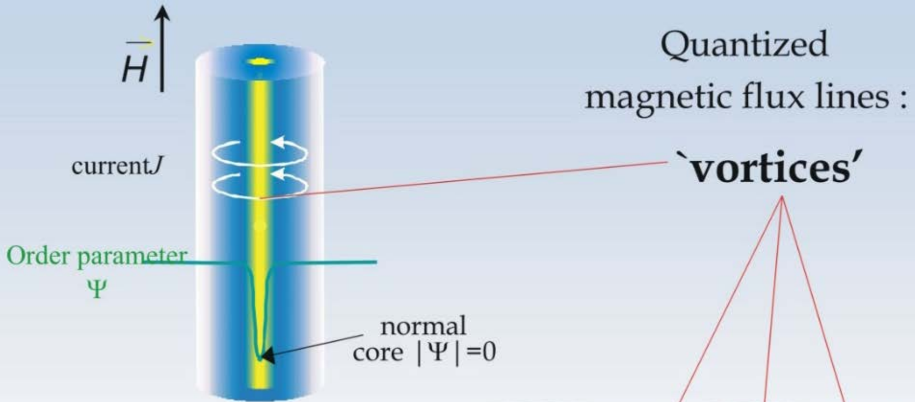
Vortices are experimentally visible

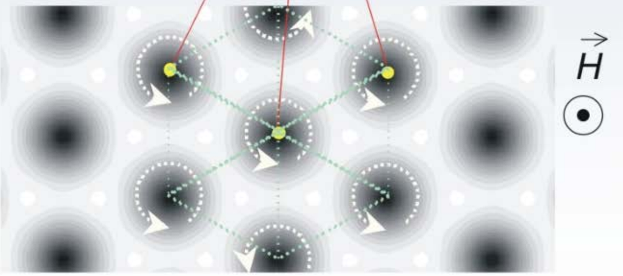


Vortex matter as a smoking gun for underlying novel phenomena

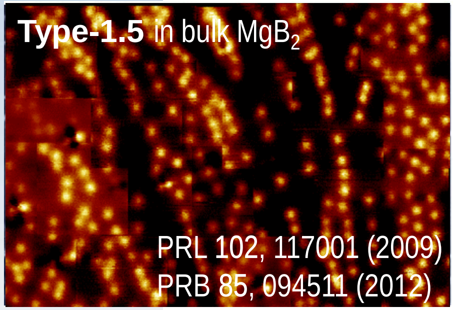


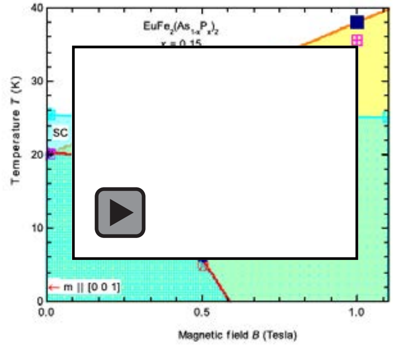


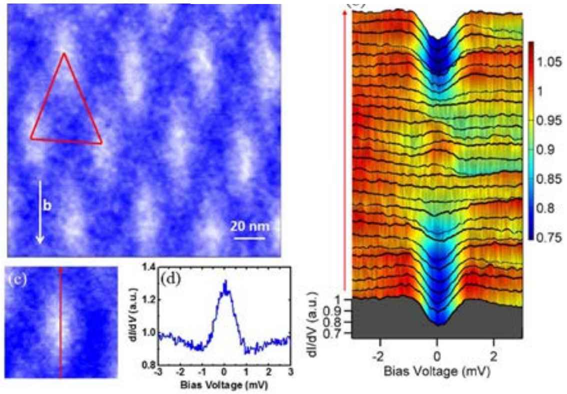




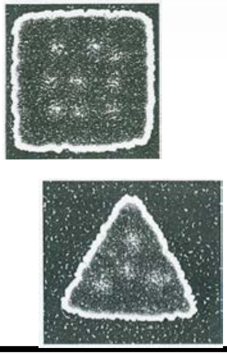
- « bulk » superconductor : triangular vortex lattice
- mesoscopic structure : size ~ ξ,λ
  Geometry dependent



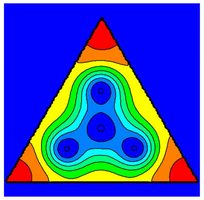


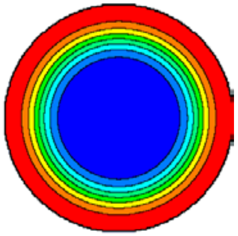


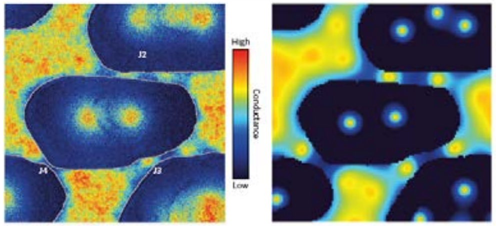
Elongated, coreless, spontaneous vortices in Fe-based superconductors, see recent works of Hai-Hu Wen, Nanjing and Hanaguri, RIKEN



Images:
I. Grigorieva







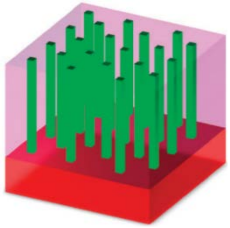
Nature Physics 11, 332 (2015)

...but vortices are mobile and can ruin everything!

Vortices move – local Joule heating – quench!

Vortex pinning

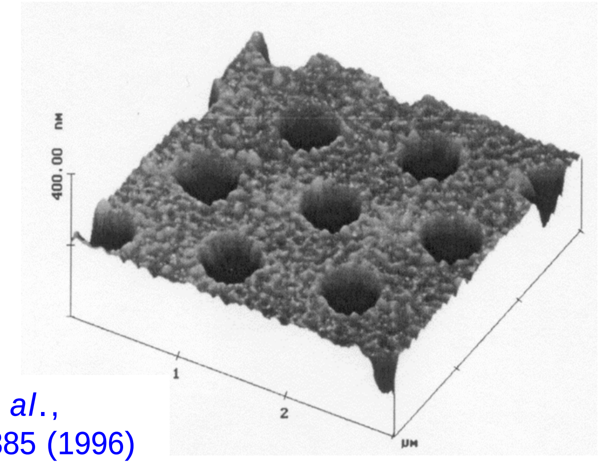
- Sample or material imperfections, possibly chemically grown
- Strategically placed artificial defects



S. Lee et al., Nature Mat.

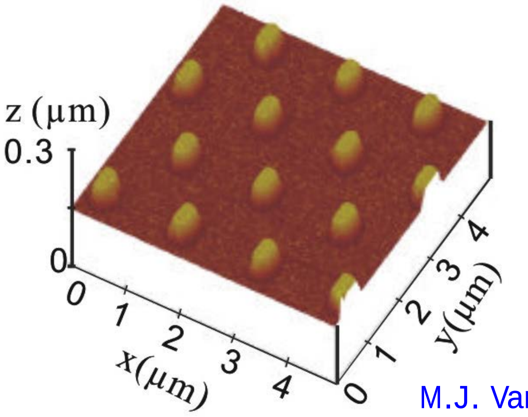
Abrikosov lattice
triangular lattice ⟷ add an underlying (permanent) lattice structure

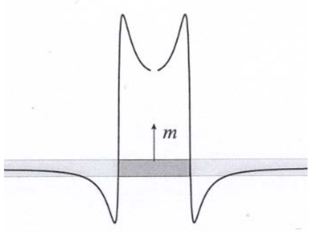
Nanostructuring of the SC



V.V. Moshchalkov *et al.*,
Phys. Rev. B. **54**, 7385 (1996)

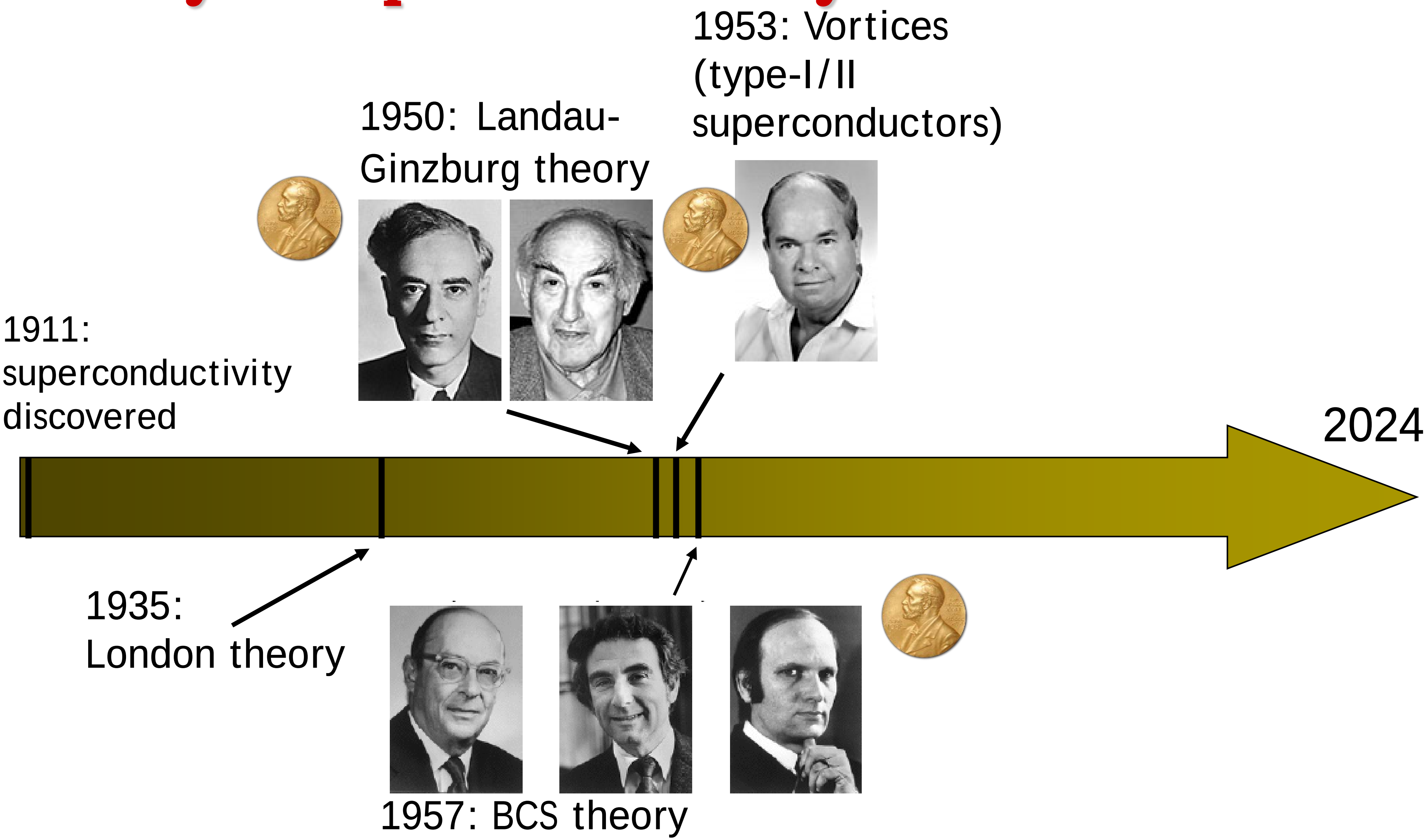
Nanostructuring of the magnetic field



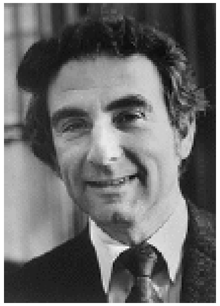


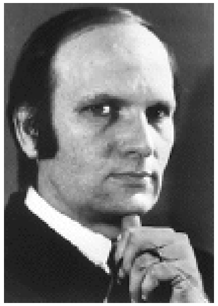
M.J. VanBael *et al.*,
Phys. Rev. B. **54**, 7385 (1996)

Theory of superconductivity



Microscopic theory of superconductivity





Bardeen Cooper Schrieffer

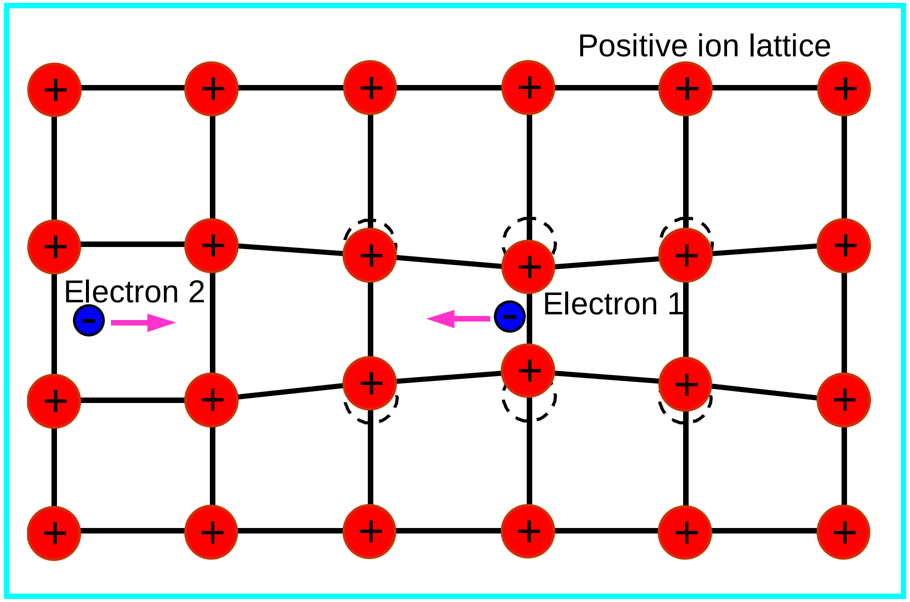
BCS theory (1957) - Nobel Prize in Physics 1972

• 'Conventional' superconductivity arises due to a condensation of electrons near the Fermi energy into *Cooper pairs.*

• Within the Bardeen, Cooper, Schrieffer (BCS) theory, Cooper pairs are bound together by the electron-phonon interaction.

• One electron distorts the ionic lattice around it, creating an area of greater positive charge density. Another electron ~ ξ_0 away is then attracted to this charge distortion (phonon).

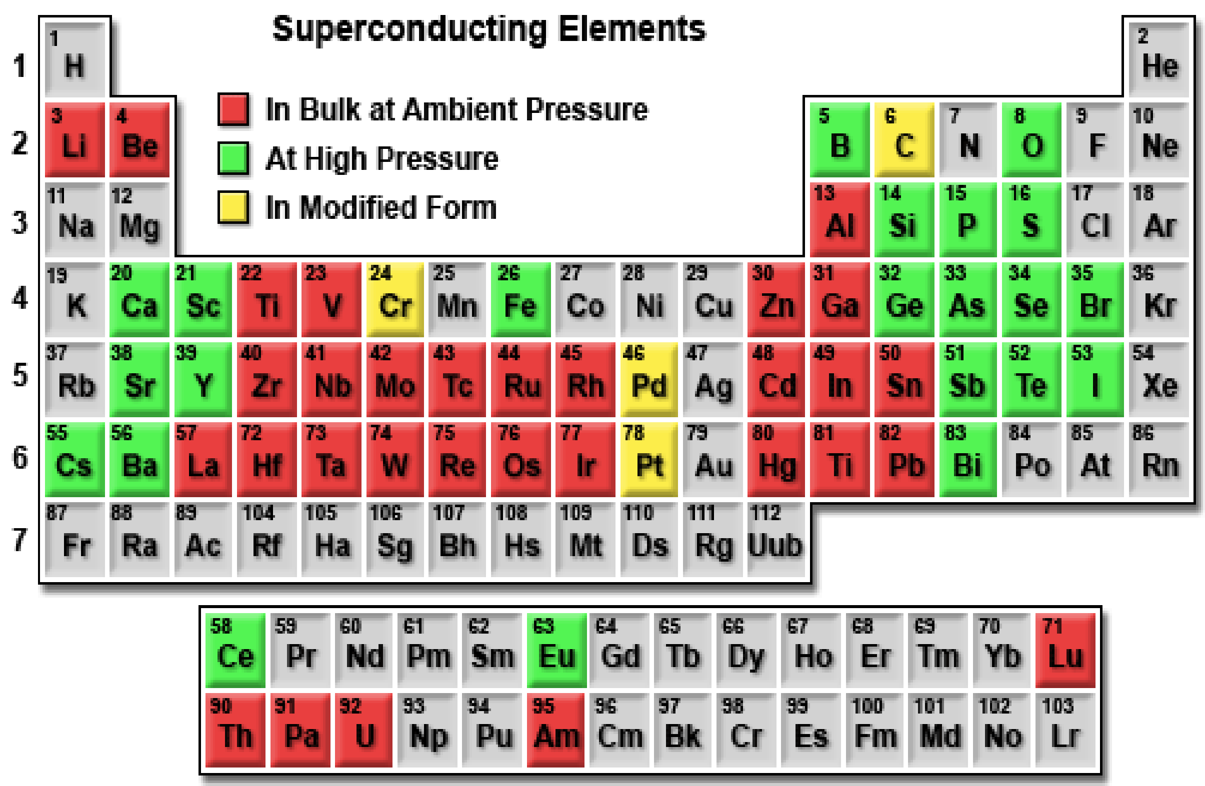
•The typical size of the pair is the *coherence length*, ξ_0 (~1-100nm).



Interestingly, BCS theory gives a very simple expression:

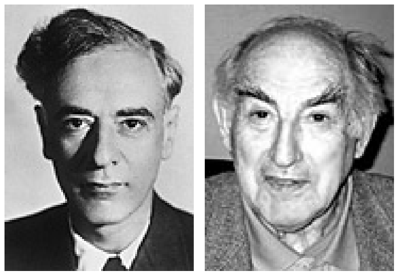
$$T_c \sim \Theta / e^{1/\gamma}$$

Poor conductors can make excellent superconductors!

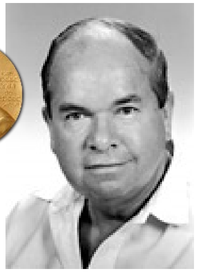


Theory of superconductivity

1950: Landau-Ginzburg theory

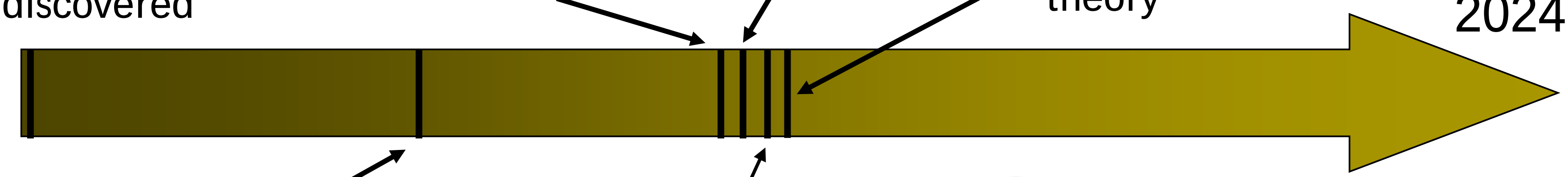


1953: Vortices (type-I/II superconductors)

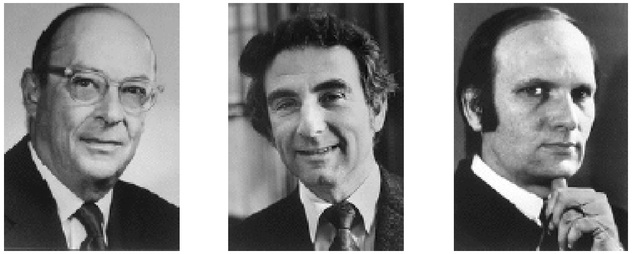


1959: Microscopic derivation of the GL theory

1911: superconductivity discovered



1935: London theory



1957: BCS theory

Gor’kov truncation

L.P. Gor’kov, Sov. Phys.-JETP 36, 1364 (1959)

The self-consistency gap equation as expansion in series over powers of Δ:

$$\Delta = \int d^3y\, K_a(r,y)\Delta(y) + \int \prod_{j=1}^{3} d^3y_j\, K_b(r,\{y\}_3)\Delta(y_1)\Delta^*(y_2)\Delta(y_3)$$

$$+\int \prod_{j=1}^{5} d^3y_j\, K_c(r,\{y\}_5)\Delta(y_1)\Delta^*(y_2)\Delta(y_3)\Delta^*(y_4)\Delta(y_5) \quad .$$

$\alpha\Delta$ $\quad$ $K\nabla^2\Delta$ $\quad$ $\beta\Delta|\Delta|^2$

1) Collecting terms $\propto \tau^{1/2}$: the above equation reduces to the critical temperature eq:

$$\Delta(\mathbf{x}) = \lambda n \mathcal{A}\, \Delta(\mathbf{x}) \qquad \frac{1}{\lambda n} = \mathcal{A} \rightarrow T_c = \frac{2e^{\Gamma}}{\pi}\hbar\omega_D e^{-1/\lambda n} \qquad \tau = 1 - \frac{T}{T_c} \ll 1$$

2) Collecting terms $\propto \tau^{3/2}$:

$$\alpha\, \Delta(\mathbf{x}) + \beta\, \Delta(\mathbf{x})|\Delta(\mathbf{x})|^2 - \mathcal{K}\, \mathbf{D}^2 \Delta(\mathbf{x}) = 0$$

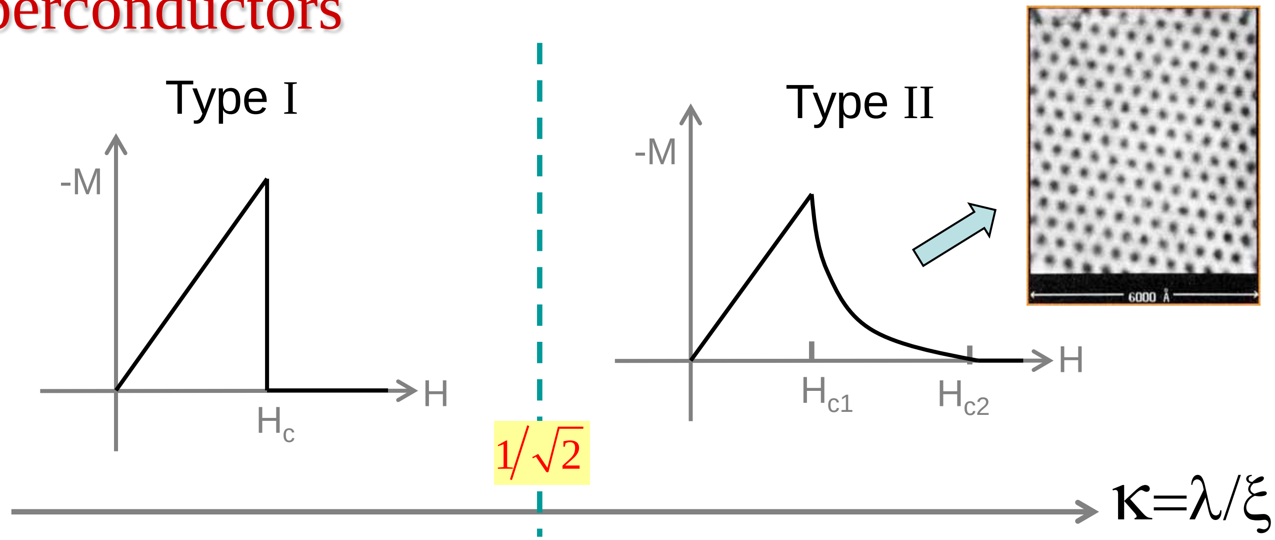
the GL equation

$$\alpha = -\tau,\ \beta = b^{(1)} = W_3^2,\ \mathcal{K} = a^{(2)} = \frac{W_3^2}{6}\hbar^2 v_i^2 \quad [W_3^2 = \frac{7\zeta(3)}{8\pi^2 T_c^2}]$$

$$\Delta(\mathbf{x}) \propto \tau^{1/2}, \quad \mathbf{D}^2 \propto \tau \quad [\xi_{\rm GL} \propto \tau^{-1/2}, \lambda_{\rm GL} \propto \tau^{-1/2}]$$

Type-I vs. type-II superconductors

'Bulk' superconductors



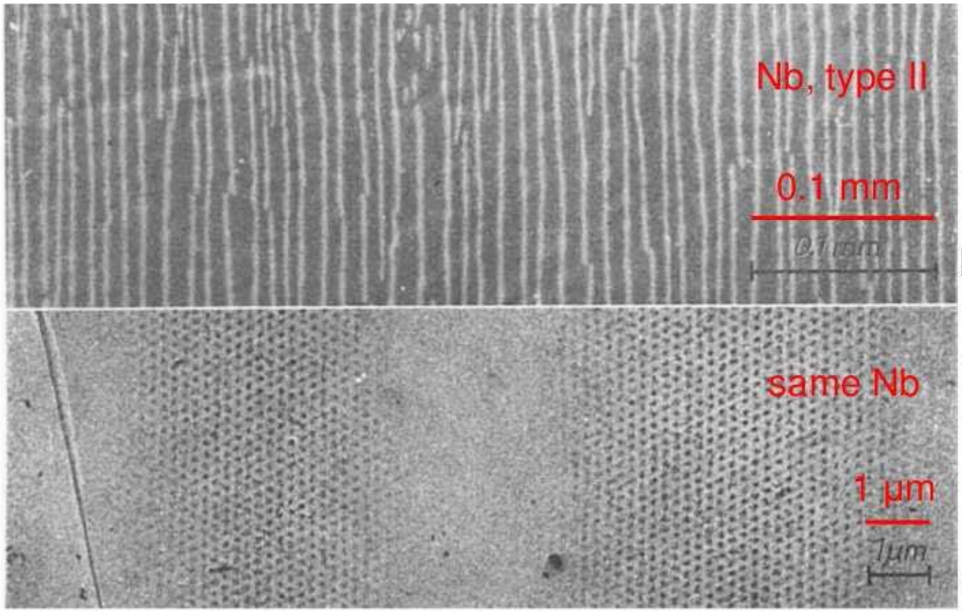
Where types I and II interchange?

- Textbooks: at critical parameter κ^* found from the conditions
  - zero surface energy for S-N interface (κ_s^*)
  - $H_c = H_{c1}$ and $H_c = H_{c2}$ (κ_1^* and κ_2^*)
  - long-range vortex interaction changes sign (κ_{li}^*)
- GL theory: $\kappa_s^* = \kappa_1^* = \kappa_2^* = \kappa_{li}^* = 1/\sqrt{2}$

Is it really that abrupt and that simple?

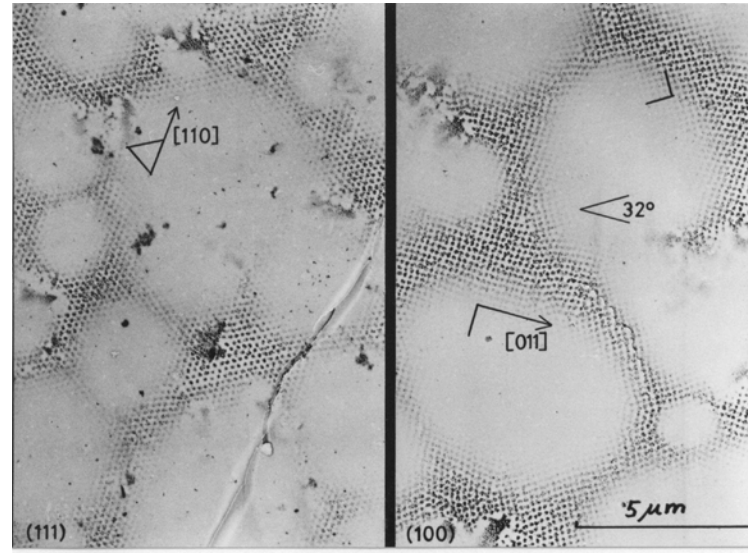
U. Essmann and H. Trauble, Phys. Lett. **24A**, 526 (1967); Sci. Am. **224**, 75 (1971).

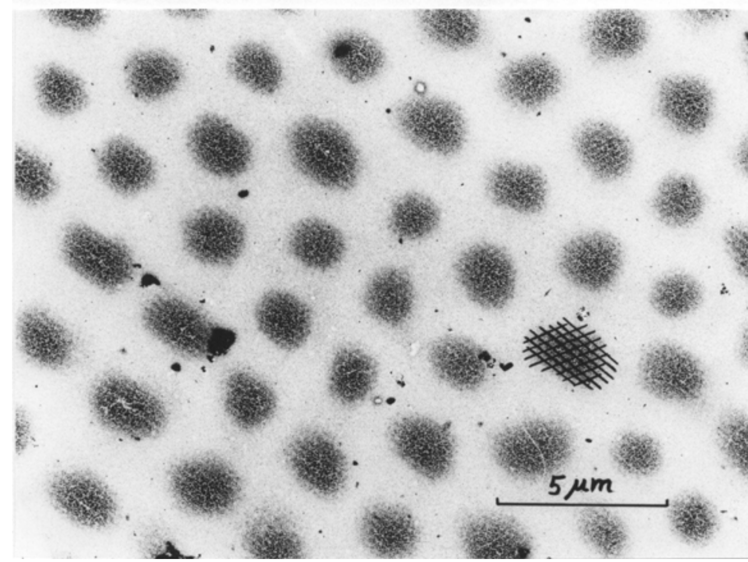
No, there is type-II/1



Top: type-II superconductor, Niobium disc, d = 40 μm, D = 4 mm, Ba = 74 mT. Optical microscope. Bottom: electron microscope.

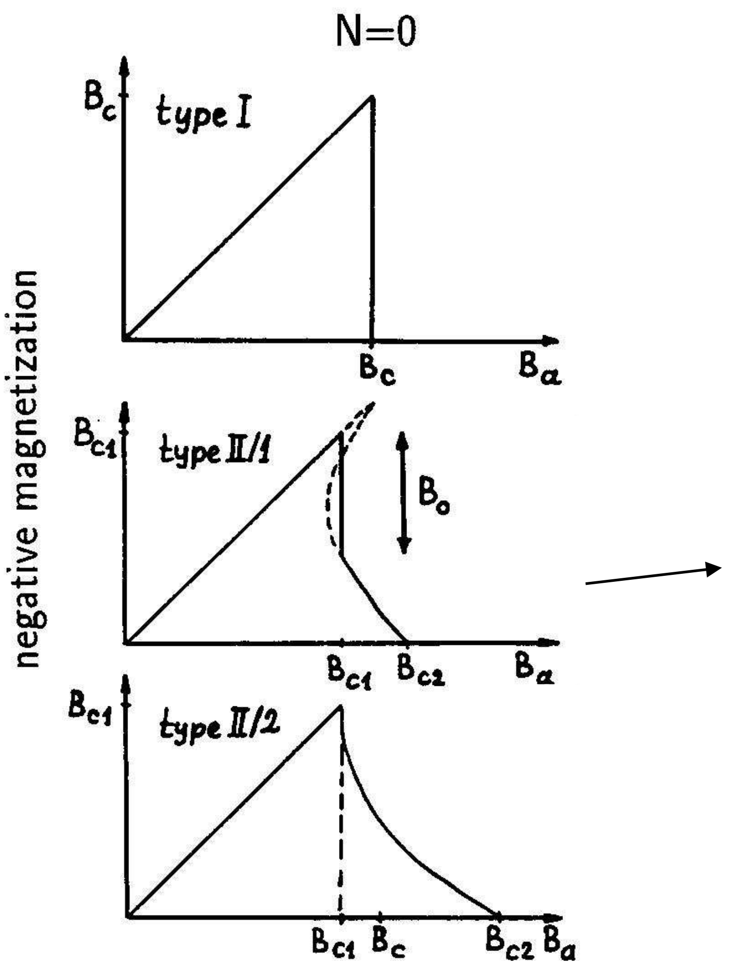
$\kappa \approx 0.75$





Top: High-purity Nb disks 1 mm thick, 4 mm diameter, of different crystallographic orientations [110] and [011], at T = 1.2 K and Ba = 800 Gauss (Bc1 = 1400 Gauss). *Bottom:* High-purity Nb foil 0.16 mm thick at T = 1.2 K and Ba = 173 Gauss. Round islands of vortex lattice embedded in a Meissner phase. (Courtesy U. Essmann)

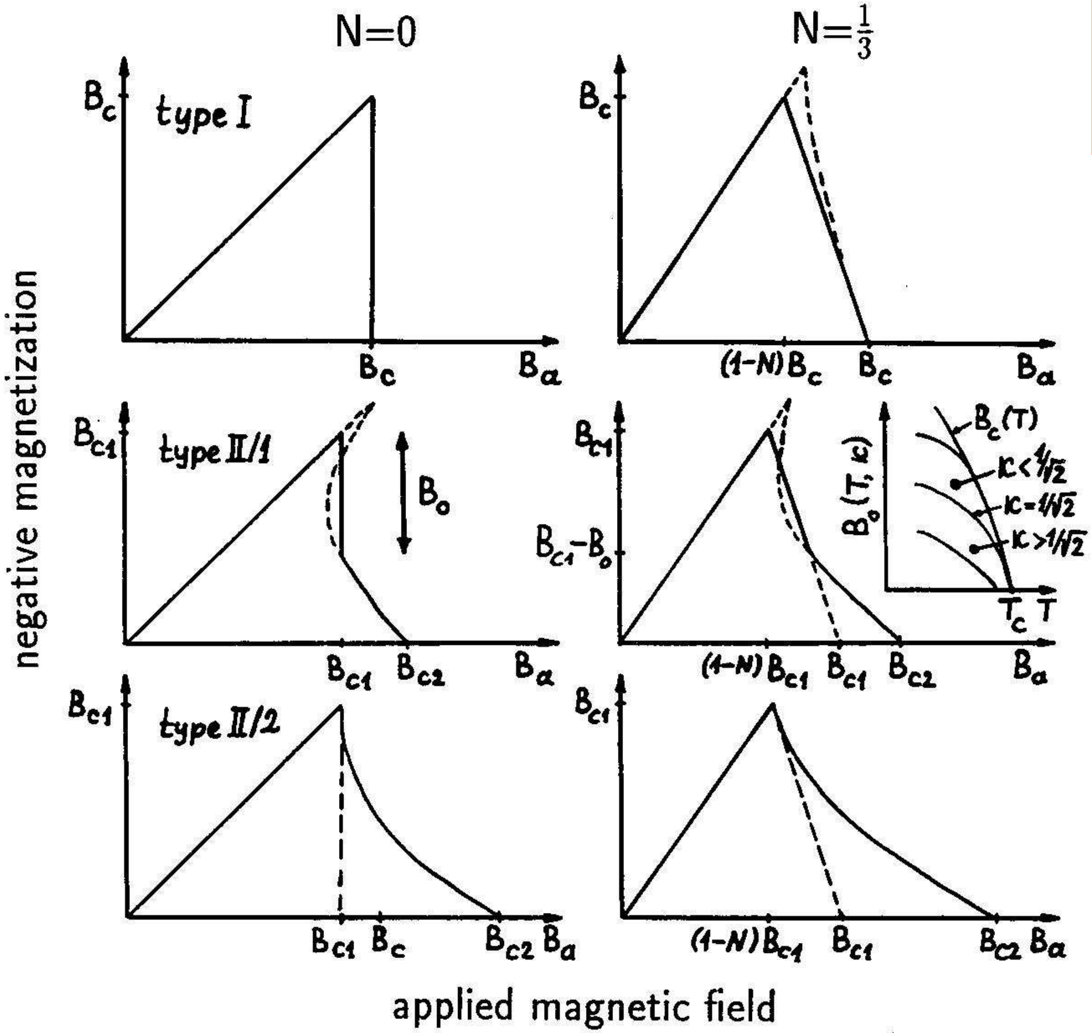
Types of superconductivity

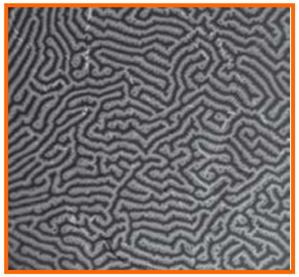


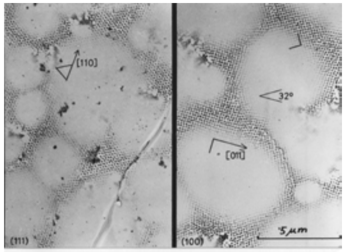
The long-range attraction between vortices in type-II/1 appears due to nonlocal effects, and not due to any special relation between characteristic lengths in the GL theory.

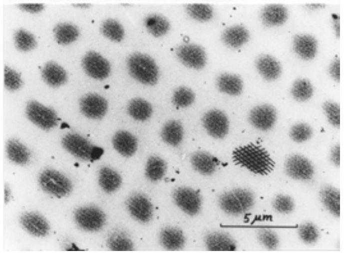
Being local, Ginzburg-Landau theory cannot capture this.

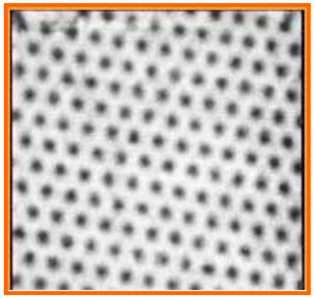
Types of superconductivity











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$$+\int\prod_{j=1}^{5} d^3y_j\; K_c(r,\{y\}_5)\Delta(y_1)\Delta^*(y_2)\Delta(y_3)\Delta^*(y_4)\Delta(y_5) \;\cdot$$

$\alpha\Delta$ $\quad$ $\mathrm{K}\nabla^2\Delta$ $\quad$ $\beta\Delta|\Delta|^2$

1) Collecting terms $\propto \tau^{1/2}$: the above equation reduces to the critical temperature eq:

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2) Collecting terms $\propto \tau^{3/2}$:

$$\alpha\,\Delta(\mathbf{x}) + \beta\,\Delta(\mathbf{x})|\Delta(\mathbf{x})|^2 - \mathcal{K}\,\mathbf{D}^2\Delta(\mathbf{x}) = 0$$

the GL equation

$$\alpha = -\tau,\; \beta = b^{(1)} = W_3^2,\; \mathcal{K} = a^{(2)} = \frac{W_3^2}{6}\hbar^2 v_i^2 \quad [W_3^2 = \frac{7\zeta(3)}{8\pi^2 T_c^2}]$$

$$\Delta(\mathbf{x}) \propto \tau^{1/2}, \quad \mathbf{D}^2 \propto \tau \quad [\xi_{\rm GL} \propto \tau^{-1/2}, \lambda_{\rm GL} \propto \tau^{-1/2}]$$

Extended GL theory

$$\Delta_i(\mathbf{x}) = \Delta_i^{(0)}(\mathbf{x}) + \Delta_i^{(1)}(\mathbf{x}), \qquad \Delta_i^{(0)}(\mathbf{x}) \propto \tau^{1/2},\ \Delta_i^{(1)}(\mathbf{x}) \propto \tau^{3/2}$$

1) $$\alpha\Delta_i^{(0)} + \beta_i[\Delta_i^{(0)}]^3 - K\nabla^2\Delta_i^{(0)} = 0, \longrightarrow \Delta_1^{(0)}(\mathbf{x})/\Delta_2^{(0)}(\mathbf{x}) = \text{const}$$

2) $$\Delta_i^{(1)}(\alpha + 3\beta_i[\Delta_i^{(0)}]^2) - K\nabla^2\Delta_i^{(1)} = F(\Delta_i^{(0)}) + F_i(\Delta_i^{(0)}),$$

$$F(\varphi) = \sigma\varphi + S\nabla^2\varphi + Y\nabla^2(\nabla^2\varphi),$$

$$F_i(\varphi) = \rho_i\varphi^3 + \chi_i\varphi^5 + U_i\varphi\nabla\cdot(\varphi\nabla\varphi) + V_i\nabla^2\varphi^3 + Z_i\varphi^2\nabla^2\varphi.$$

$$\sigma = -\left(\frac{a_1a_2}{\gamma} - \gamma\right)_{\tau^2}, \quad S = \left(\frac{\mathcal{K}_1a_2 + \mathcal{K}_2a_1}{\gamma}\right)_{\tau}, \quad Y = \left(\frac{\mathcal{Q}_1a_2 + \mathcal{Q}_2a_1 - \mathcal{K}_1\mathcal{K}_2}{\gamma}\right)_{\tau^0},$$

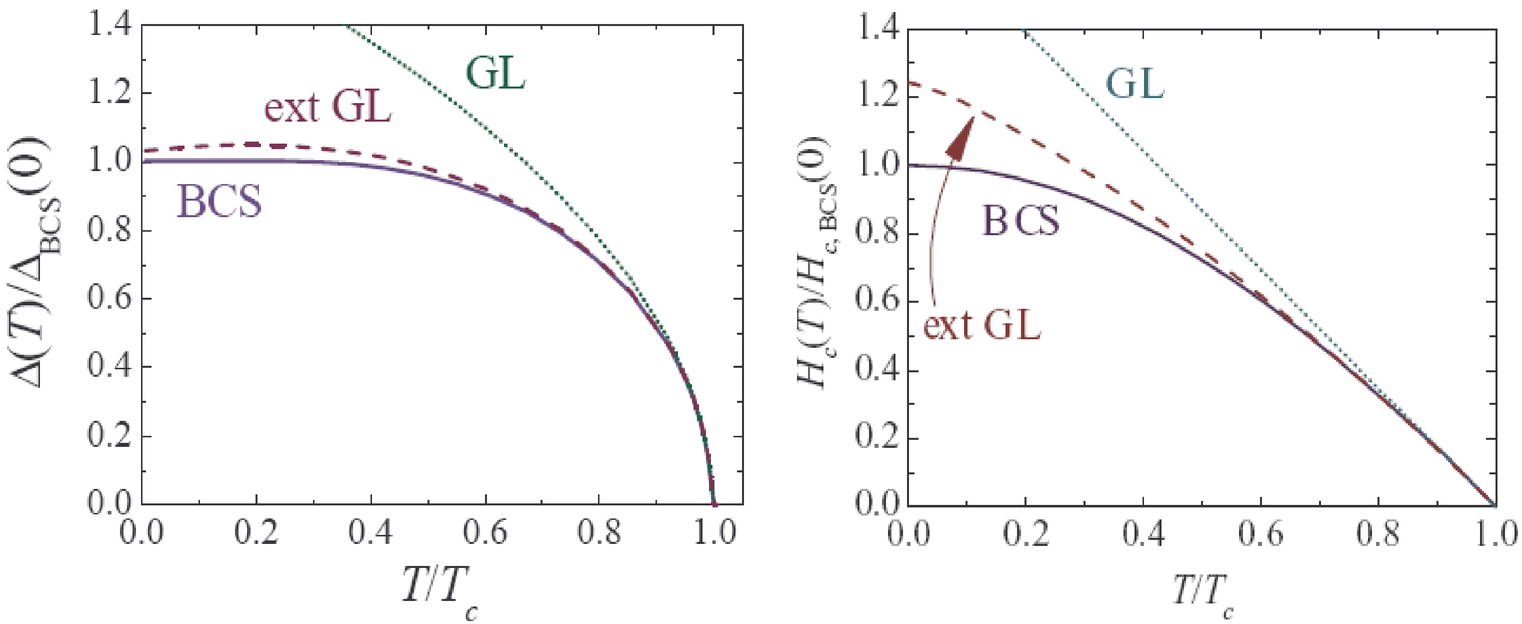
$$\rho_1 = -\left(\frac{b_1a_2 + a_1^3b_2/\gamma^2}{\gamma}\right)_{\tau}, \quad \chi_1 = \left(\frac{c_1a_2 - 3a_1^2b_1b_2/\gamma^2 + a_1^5c_2/\gamma^4}{\gamma}\right)_{\tau^0},$$

$$U_1 = -\left(\frac{\mathcal{L}_1a_2 + a_1^3\mathcal{L}_2/\gamma^2}{\gamma}\right)_{\tau^0}, \quad V_1 = \left(\frac{b_1\mathcal{K}_2}{\gamma}\right)_{\tau^0}, Z_1 = 3\left(\frac{a_1^2\mathcal{K}_1b_2}{\gamma^3}\right)_{\tau^0}$$

How well does Extended GL work?

A.V. Vagov, A.A. Shanenko, M.V. Milošević, *et al.*, Phys. Rev. B **85**, 014502 (2012)

Ripley's *Believe it or not* agreement with BCS at lower temperatures



The current-field relation becomes non-local in the next-to-leading order!

Can EGL then capture physics outside type-I/type-II?

Opening of the Bogomolnyi domain

Conditions around the Bogomolnyi point:

$$a = \frac{a_1}{S} + S a_2,\ b = \frac{b_1}{S^2} + S^2 b_2,\ \mathcal{K} = \frac{\mathcal{K}_1}{S} + S\mathcal{K}_2,$$
$$\alpha = \frac{a_1}{S} - S a_2,\ \beta = \frac{b_1}{S^2} - S^2 b_2,\ \Gamma = \frac{\mathcal{K}_1}{S} - S\mathcal{K}_2,$$
$$c = \frac{c_1}{S^3} + S^3 c_2,\ \mathcal{Q} = \frac{\mathcal{Q}_1}{S} + S\mathcal{Q}_2,\ \mathcal{L} = \frac{\mathcal{L}_1}{S^2} + S^2\mathcal{L}_2,$$
$$\tilde{c} = \frac{ca}{3b^2},\ \tilde{\mathcal{Q}} = \frac{\mathcal{Q}a}{\mathcal{K}^2},\ \tilde{\mathcal{L}} = \frac{\mathcal{L}a}{b\mathcal{K}},\ \tilde{G} = \frac{Ga}{4g_{12}},\ G = g_{11}g_{22} - g_{12}^2,$$
$$\tilde{\alpha} = \frac{\alpha}{a} - \frac{\Gamma}{\mathcal{K}},\ \tilde{\beta} = \frac{\beta}{a} - \frac{\Gamma}{\mathcal{K}},\ \chi = N_2(0)/N_1(0)$$
$$S = \frac{1}{2\lambda_{12}}\left[\lambda_{22} - \frac{\lambda_{11}}{\chi} + \sqrt{\left(\lambda_{22} - \frac{\lambda_{11}}{\chi}\right)^2 + 4\frac{\lambda_{12}^2}{\chi}}\right]$$

$\kappa_2^* \neq \kappa_s^* \neq \kappa_1^* \neq \kappa_{li}^*$ at $T < T_c$

$H_{c2} = H_c$

$$\kappa_2^* = \frac{1}{\sqrt{2}}\left\{1 + \tau\left[1 - \tilde{c} + 2\tilde{\mathcal{Q}} - \tilde{G}\tilde{\beta}(\tilde{\beta} - 2\tilde{\alpha})\right]\right\}$$

$H_{c1} = H_c$

$E_s = 0$

κ^*_{li}

$$\kappa^* = \frac{1}{\sqrt{2}}\left\{1 + \tau\left[1 - \tilde{c} + 2\tilde{\mathcal{Q}} + \tilde{G}\tilde{\beta}(2\tilde{\alpha} - \tilde{\beta}) + \frac{J}{I}\left(2\tilde{\mathcal{L}} - \tilde{c} - \frac{5}{3}\tilde{\mathcal{Q}} - \tilde{G}\tilde{\beta}^2\right)\right]\right\}$$
$$I = \int d^3r\, |\psi|^2(1 - |\psi|^2),\ J = \int d^3r\, |\psi|^4(1 - |\psi|^2)$$

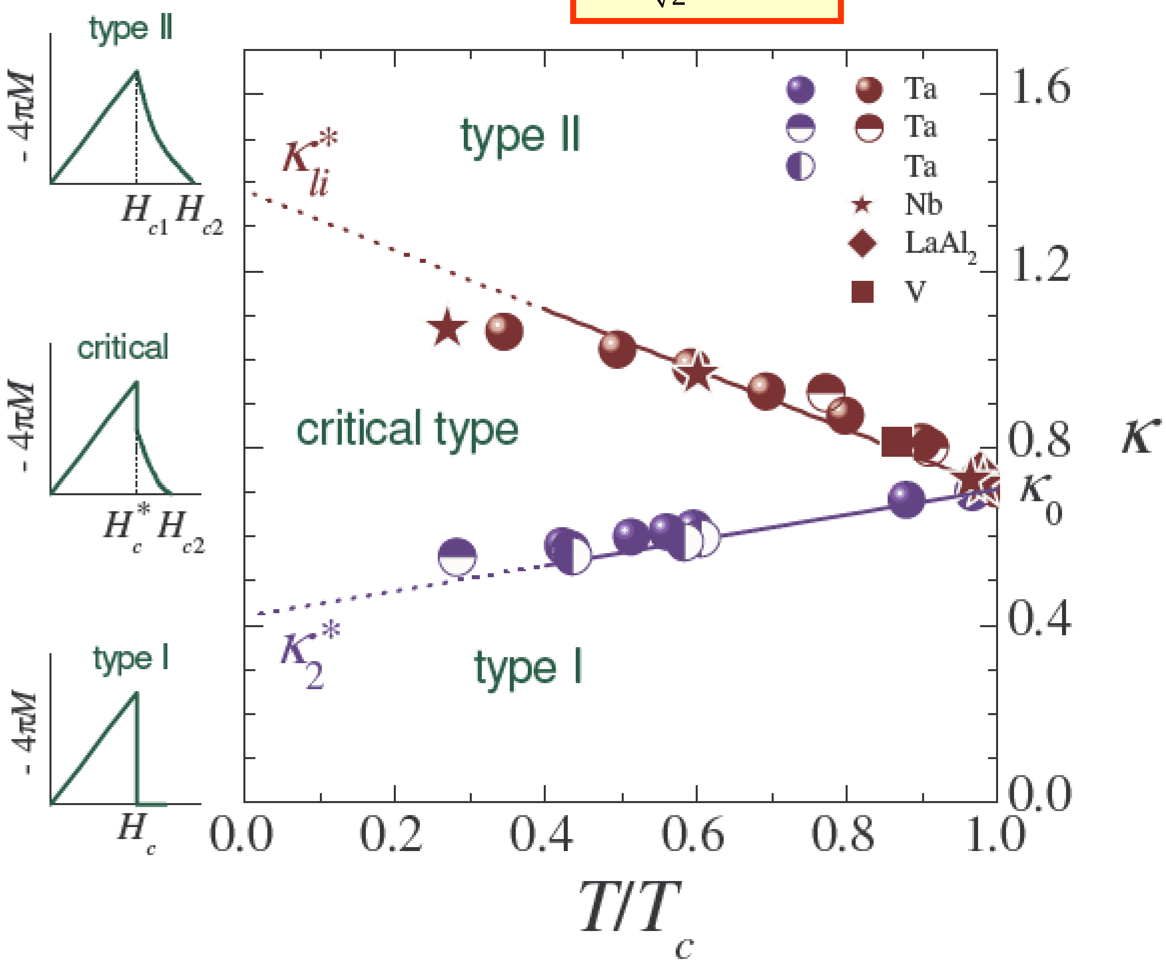
$$\kappa_2^* = \frac{1}{\sqrt{2}}(1 - 0.407\tau)$$
$$\kappa_1^* = \frac{1}{\sqrt{2}}(1 - 0.093\tau)$$
$$\kappa_s^* = \frac{1}{\sqrt{2}}(1 - 0.027\tau)$$
$$\kappa_{li}^* = \frac{1}{\sqrt{2}}(1 + 0.67\tau)$$

κ^*_{li} is calculated by substituting the long range asymptotic of the single vortex solution into the expression for the free energy (J/I=2) and then finding the value of kappa at which the long range asymptotic of the interaction changes its sign.

The critical domain

$$\kappa_2^* = \frac{1}{\sqrt{2}}(1-0.407\tau)$$
$$\kappa_1^* = \frac{1}{\sqrt{2}}(1-0.093\tau)$$
$$\kappa_s^* = \frac{1}{\sqrt{2}}(1-0.027\tau)$$
$$\kappa_{li}^* = \frac{1}{\sqrt{2}}(1+0.67\tau)$$

J. Auer and H. Ullmaier, Phys. Rev. B **7**,136 (1973)
H. W. Weber et al., Physica C **161**, 272 (1989).
F.M. Sauerzopf et al., J. Low Temp. Phys. **66**, 191 (1987)
J. F. Sporna et al., J. Low Temp. Phys. **37**, 639 (1979).



J. Auer and H. Ullmaier, Phys. Rev. B **7**, 136 (1973)

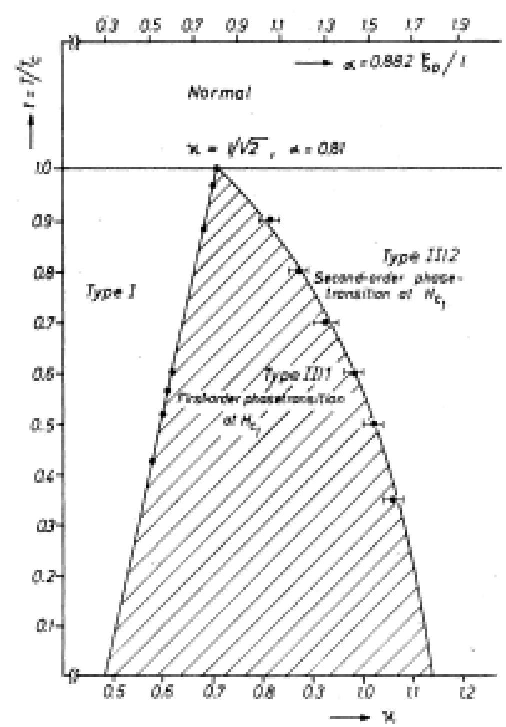
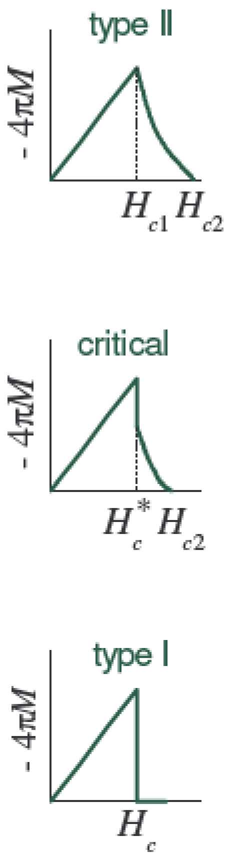
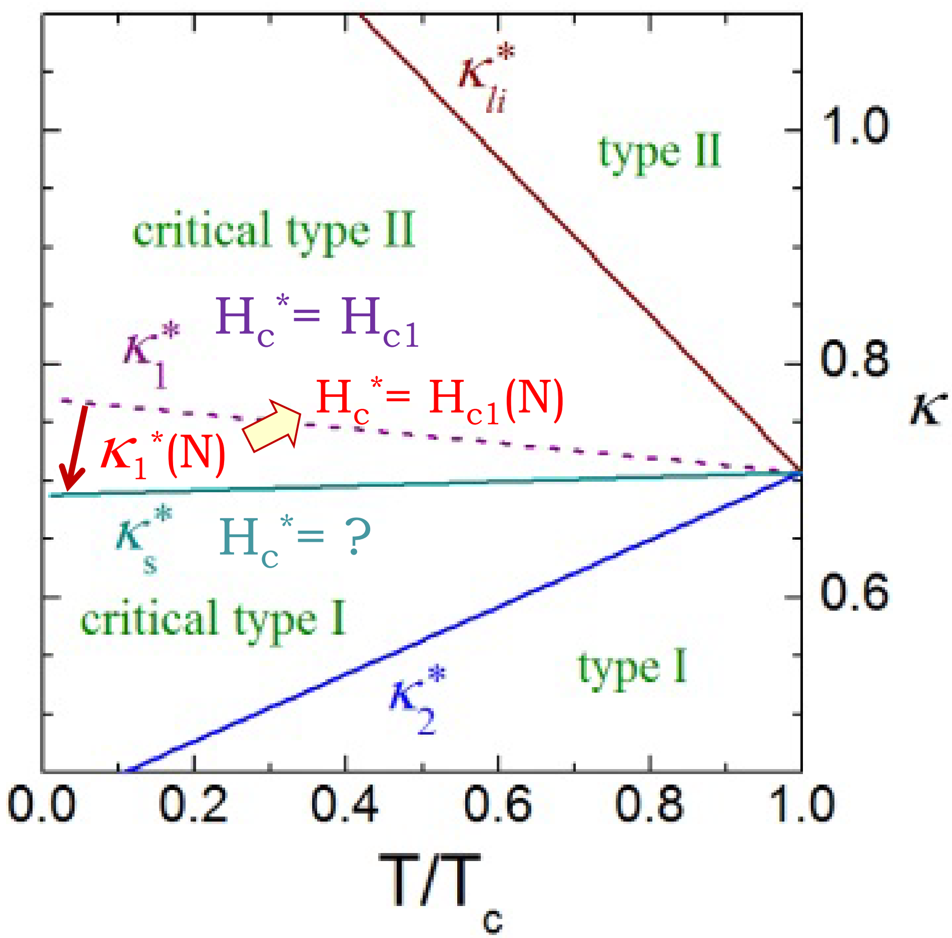


FIG. 8. Phase diagram of the magnetic behavior for the TaN system is shown. The Ginzburg–Landau parameter κ (lower abscissa) and the impurity parameter α (upper abscissa) are proportional to the amount of dissolved nitrogen.

Subdivision of the critical domain





J. Auer and H. Ullmaier, Phys. Rev. B **7**, 136 (1973)

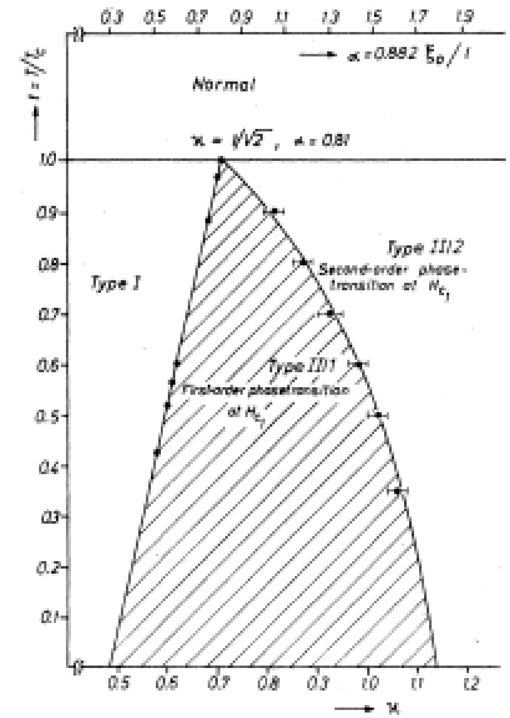
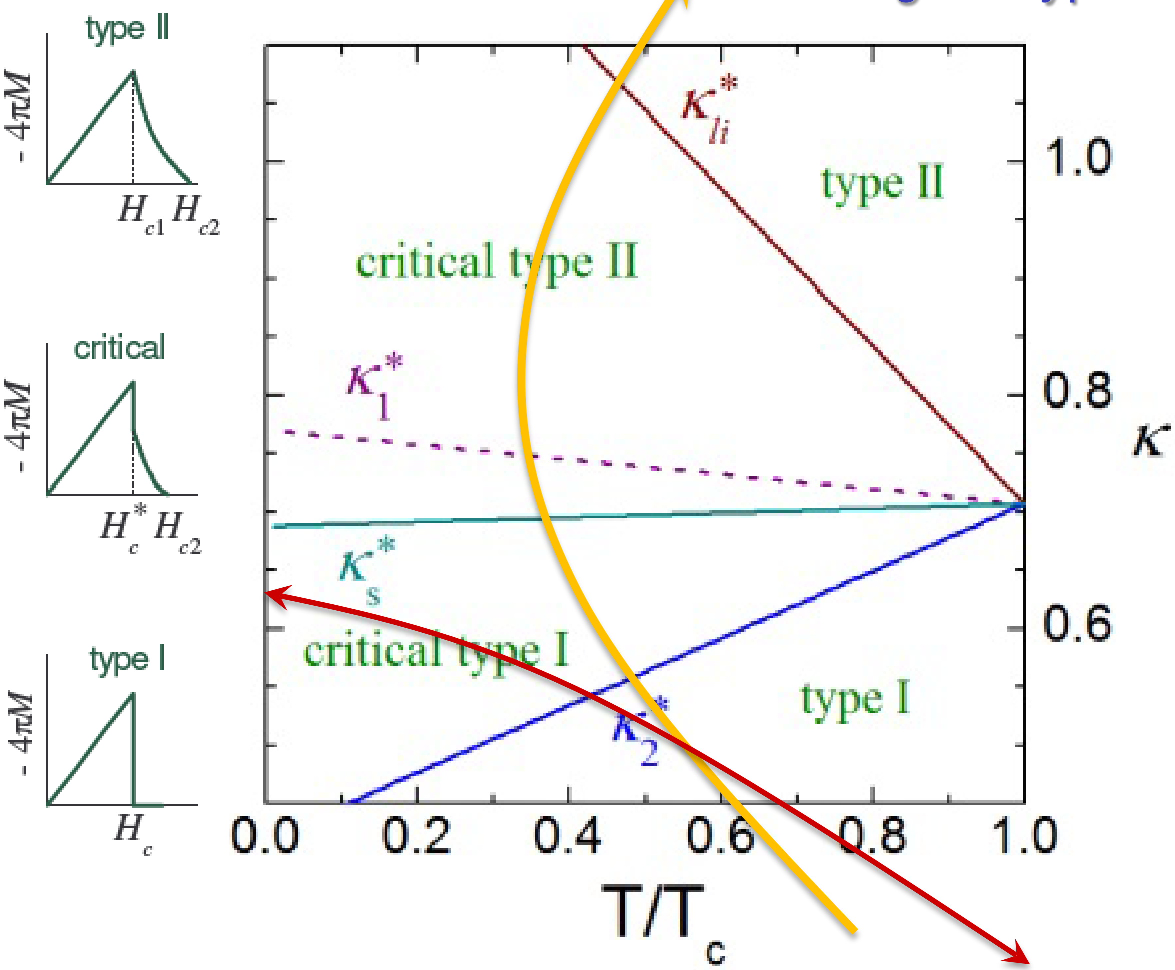


FIG. 8. Phase diagram of the magnetic behavior for the TaN system is shown. The Ginzburg–Landau parameter κ (lower abscissa) and the impurity parameter α (upper abscissa) are proportional to the amount of dissolved nitrogen.

Sampling of the critical domain

Thinning the type-I material



κ(T) for bulk Pb

J. Auer and H. Ullmaier, Phys. Rev. B **7**, 136 (1973)

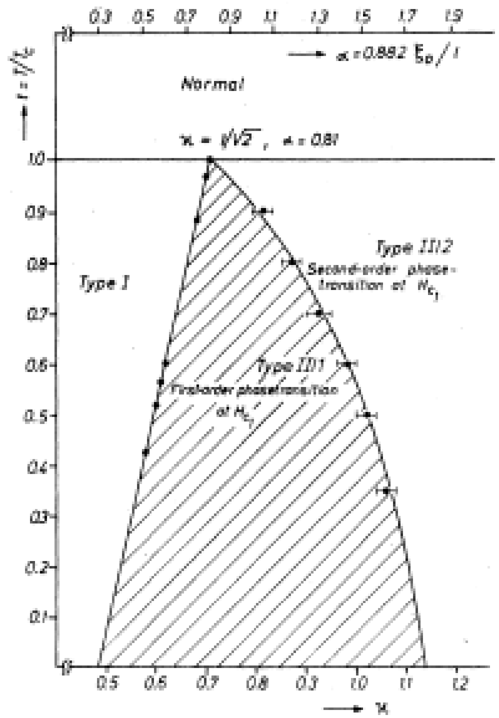


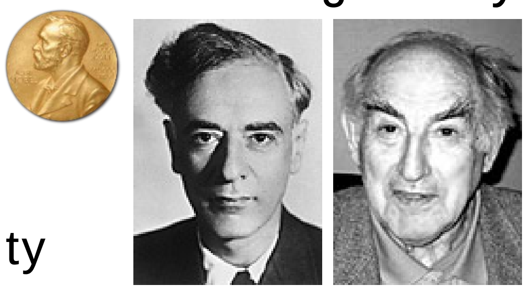
FIG. 8. Phase diagram of the magnetic behavior for the TaN system is shown. The Ginzburg–Landau parameter κ (lower abscissa) and the impurity parameter α (upper abscissa) are proportional to the amount of dissolved nitrogen.

Theory of superconductivity

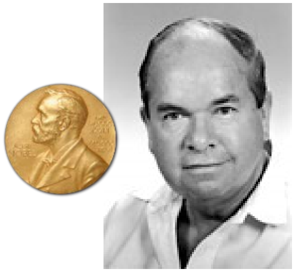
1911: superconductivity discovered

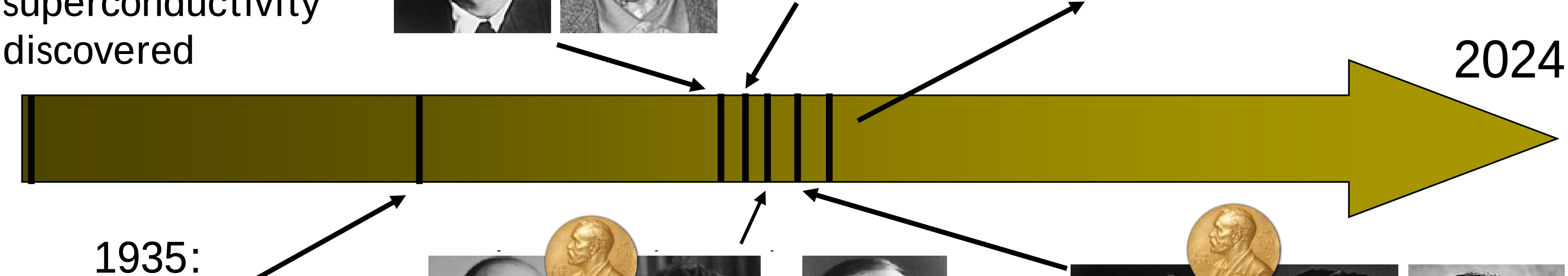
1935: London theory

1950: Landau-Ginzburg theory



1953: Vortices (type-I/II superconductors)





1957: BCS theory

1962: Josephson effect

1969: “A nail in the coffin of superconductivity”

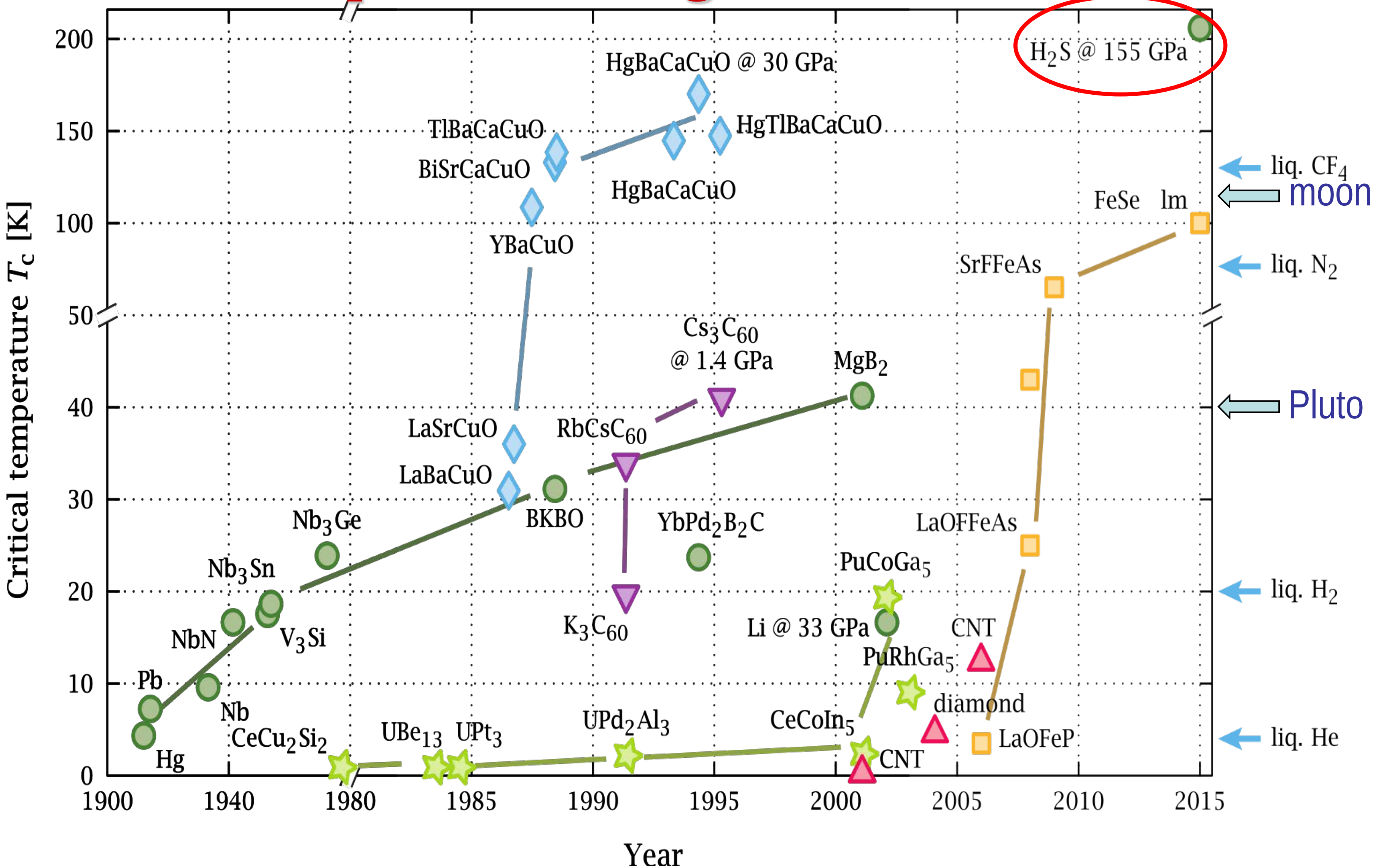
High-T_c superconductivity

1986 Bednorz and Müller – first high-T_c superconductor (perovskites (ceramic!), Ba-La-Cu-O, T_c~30K) - *Nobel prize in 1987**

Soon after, Chu *et al.* discovered YBCO, with T_c>90K, and more importantly, above the temperature of liquid nitrogen!

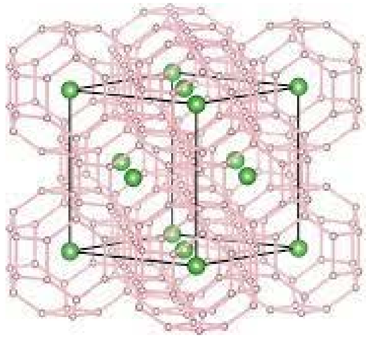
* Message to chemists: if you think you have a new (or old) material with unusual structural or chemical properties, do what Bednorz and Müller (and many others before and after) did – cool it down. For example, Claude Michel and Bernard Raveau at the University of Caen in France had made 123 stoichiometric copper-oxide perovskites in 1982, but having no cryogenic facilities at their lab or access to others elsewhere, they missed making the history.

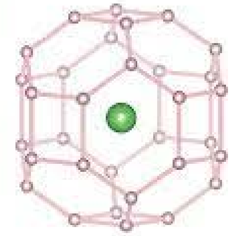
Superconducting materials



Near room-temperature superconductivity

La-superhydrides, LaH_{10} & LaH_{16}





PHYSICAL REVIEW LETTERS **122,** 027001 (2019)

Editors' Suggestion | Featured in Physics

Evidence for Superconductivity above 260 K in Lanthanum Superhydride at Megabar Pressures

Maddury Somayazulu,[1,*] Muhtar Ahart,[1] Ajay K. Mishra,[2,‡] Zachary M. Geballe,[2] Maria Baldini,[2,§] Yue Meng,[3] Viktor V. Struzhkin,[2] and Russell J. Hemley[1,†]

[1] *Institute for Materials Science and Department of Civil and Environmental Engineering, The George Washington University, Washington, DC 20052, USA*

[2] *Geophysical Laboratory, Carnegie Institution of Washington, Washington, DC 20015, USA*

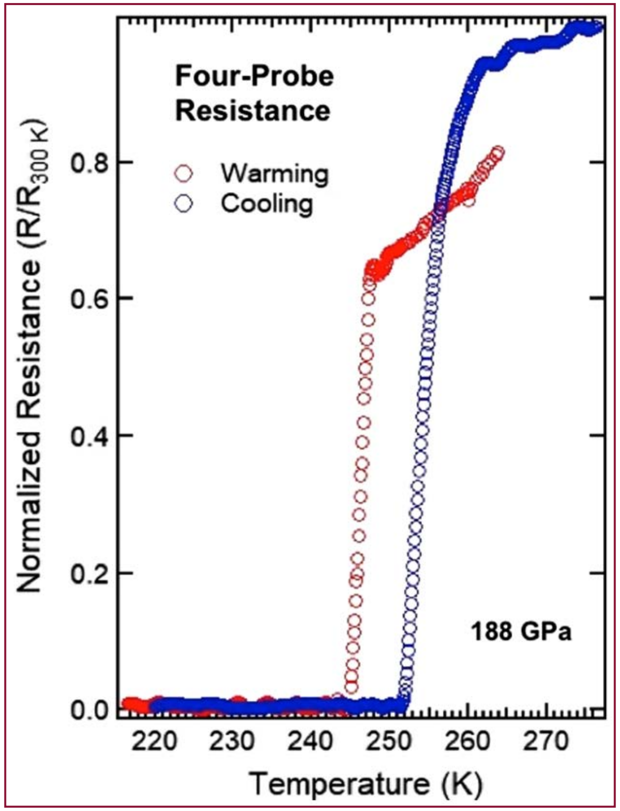
[3] *HPCAT, X-ray Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

LETTER

https://doi.org/10.1038/s41586-019-1201-8

Superconductivity at 250 K in lanthanum hydride under high pressures

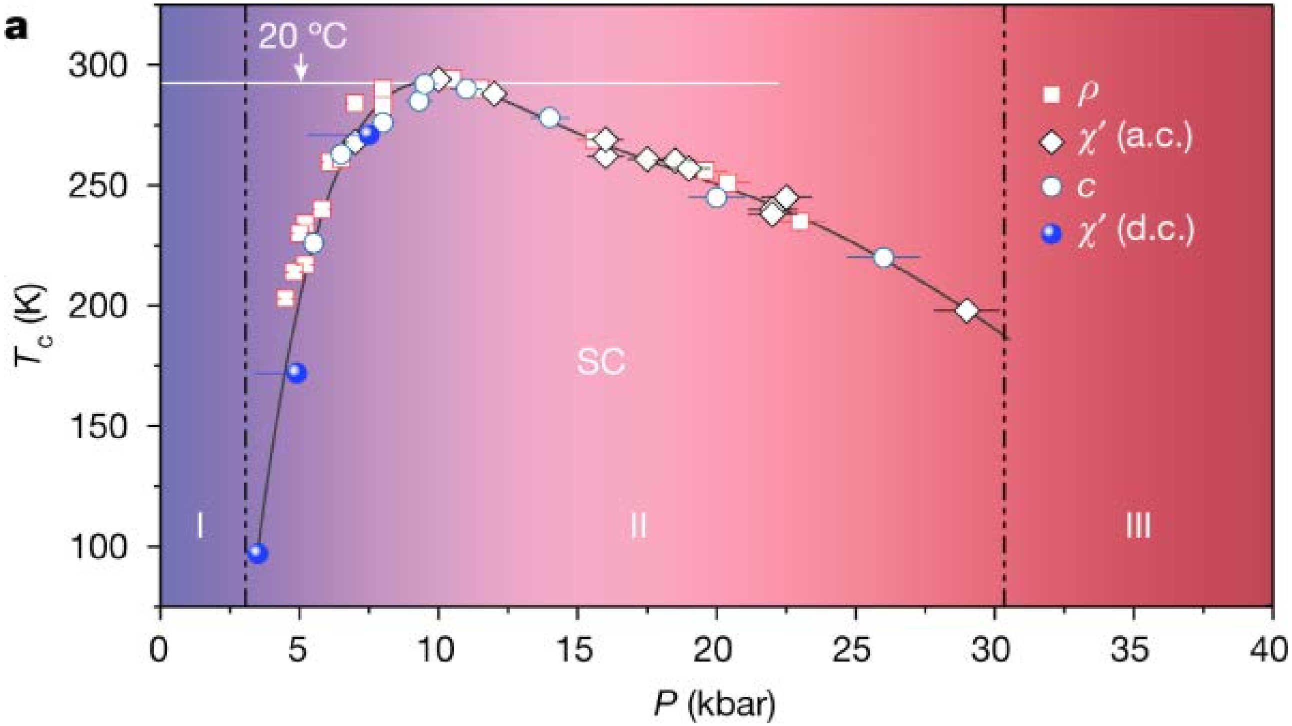
A. P. Drozdov[1,7], P. P. Kong[1,7], V. S. Minkov[1,7], S. P. Besedin[1,7], M. A. Kuzovnikov[1,6,7], S. Mozaffari[2], L. Balicas[2], F. F. Balakirev[3], D. E. Graf[2], V. B. Prakapenka[4], E. Greenberg[4], D. A. Knyazev[1], M. Tkacz[5] & M. I. Eremets[1*]

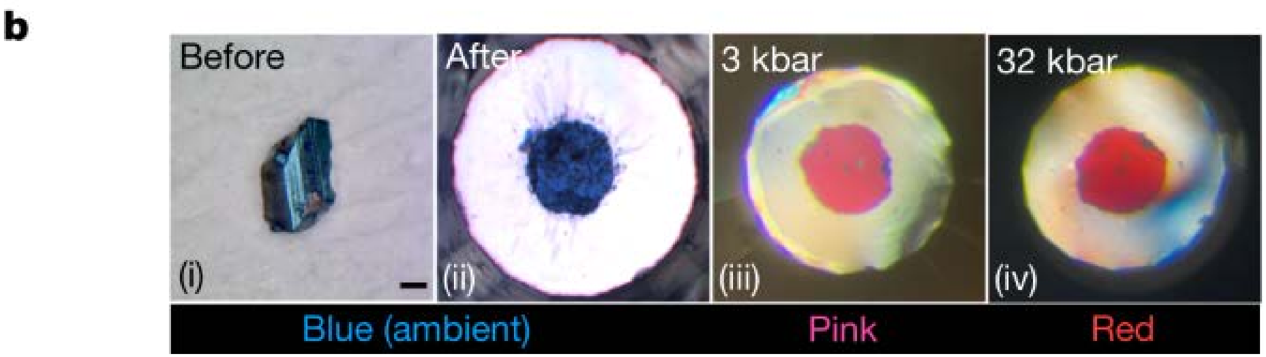


…add carbon and room-temperature superconductivity is reached!

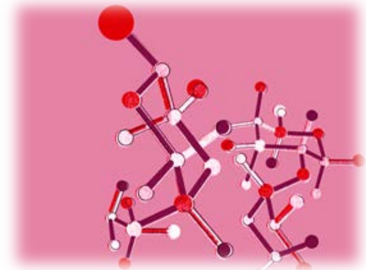
Nature (2020-2022) - retracted

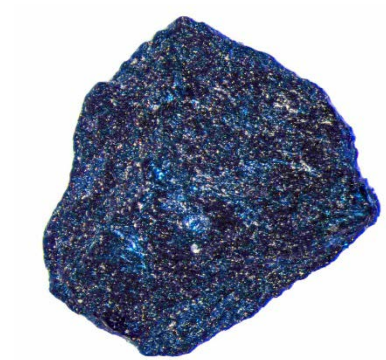
Dias returns with a vengeance [8/3/2023]





Predictive calculations could not verify it





Lutetium-hydride with ~1% nitrogen impurities

Nature 615, 244 (2023)
- retracted

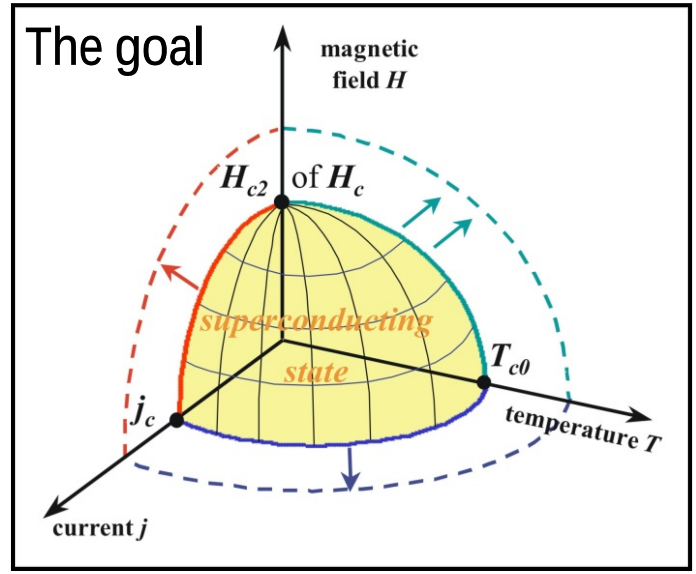
The Holy Trinity of superconductivity

materials

research

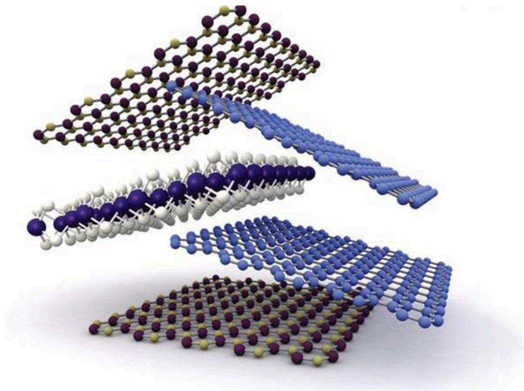
vortex matter

mechanisms

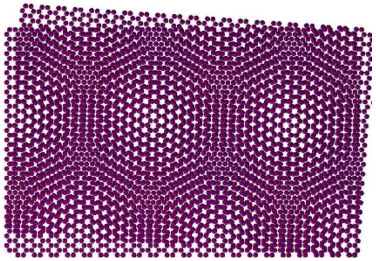


Superconductivity in 2D materials and heterostructures

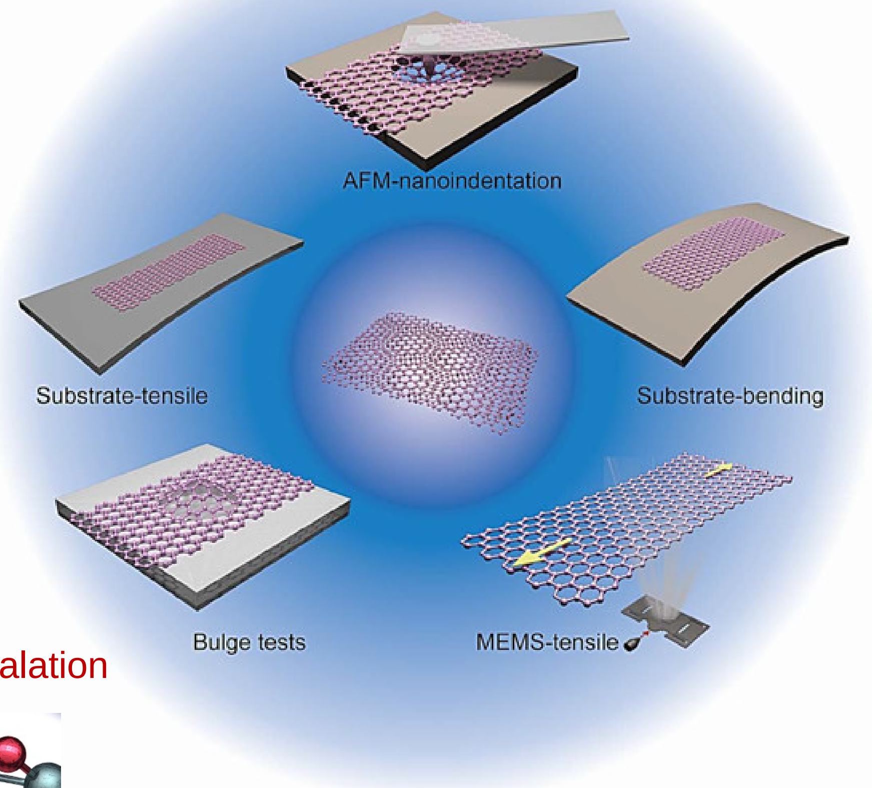
Mix&match



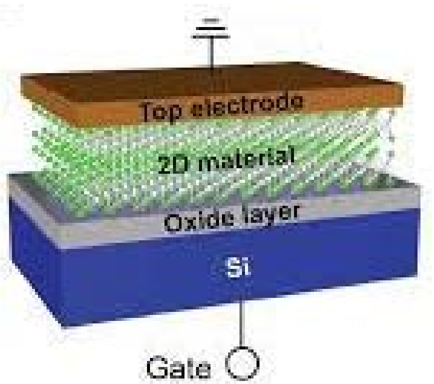
Twist



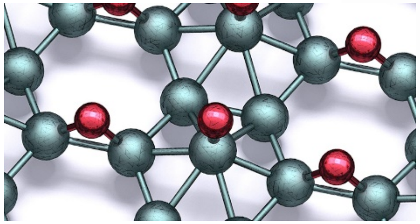
Strain



Gating

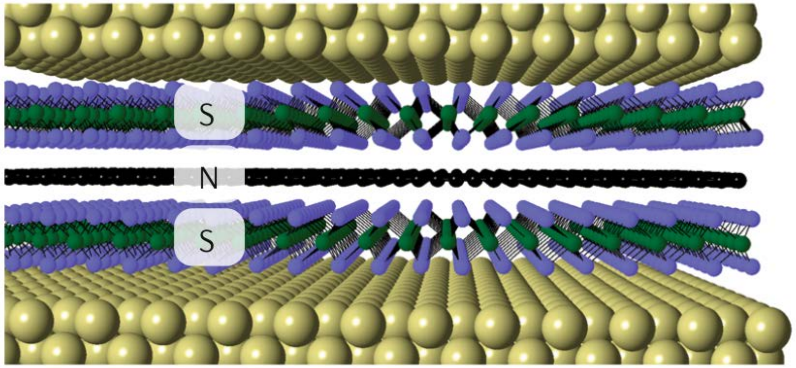


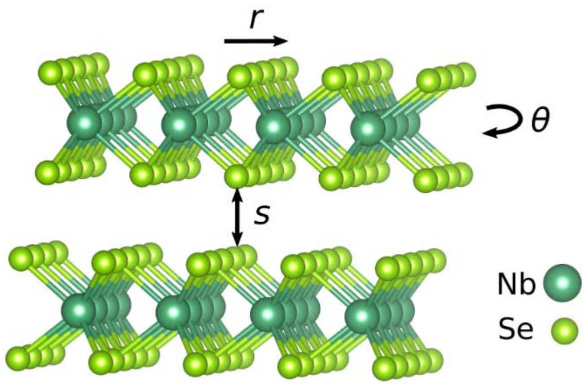
Functionalization and intercalation



⇒ ideal platform for *in silico* design of superconductors!

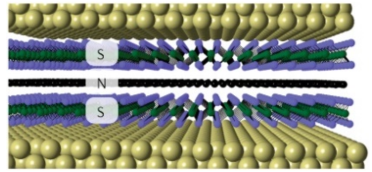
VdW Josephson junctions

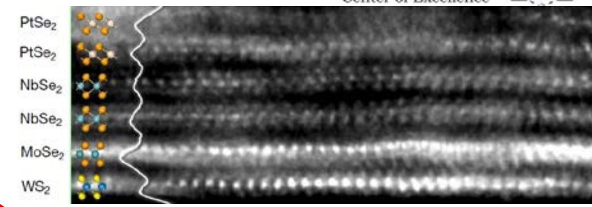


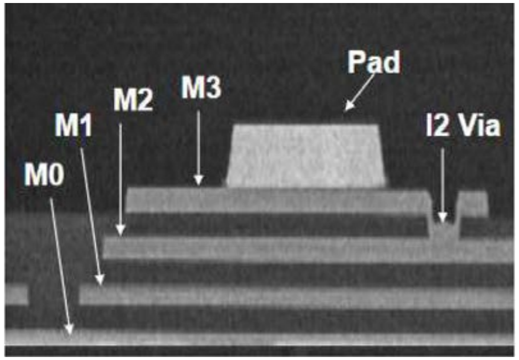


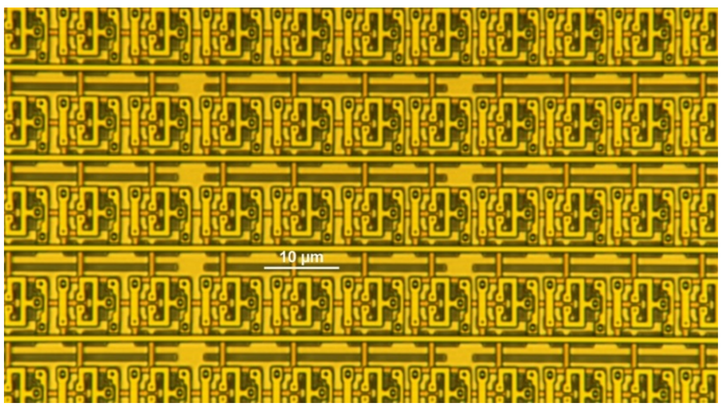
Tunable by <u>strain, gating, and twist</u>

VdW Josephson junctions









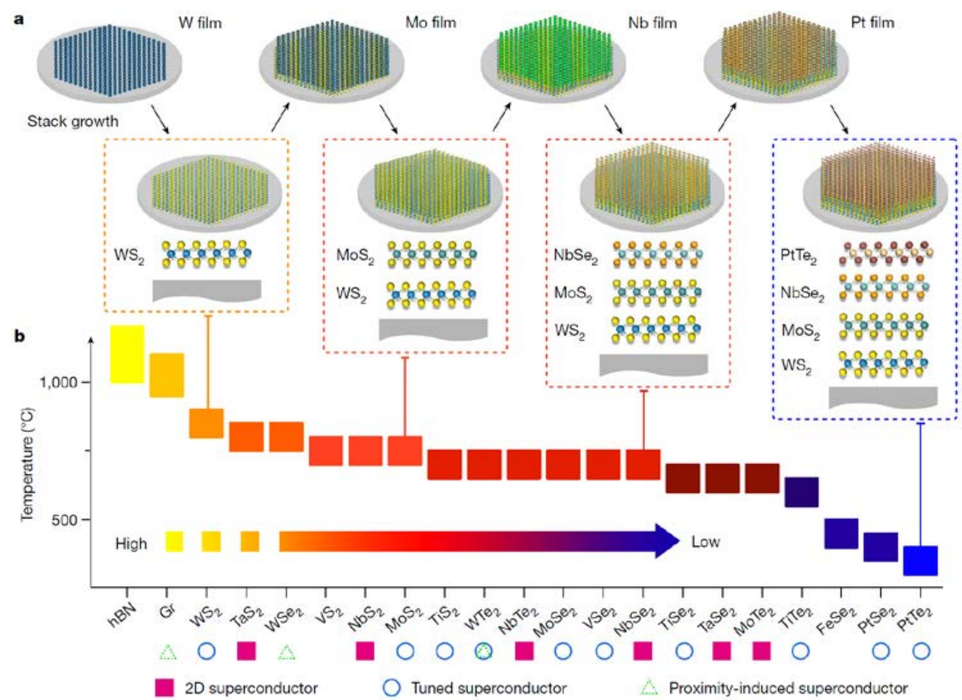
Photomicrograph of superconducting random-access memory (RAM) circuit with record density of ~4 million Josephson junctions per square centimeter, fabricated at Lincoln Laboratory, MIT, USA.

Article | Published: 06 September 2023

Stack growth of wafer-scale van der Waals superconductor heterostructures

Zhenjia Zhou, Fuchen Hou, Xianlei Huang, Gang Wang, Zihao Fu, Weilin Liu, Guowen Yuan, Xiaoxiang Xi, Jie Xu, Junhao Lin & Libo Gao

Nature (2023) | Cite this article

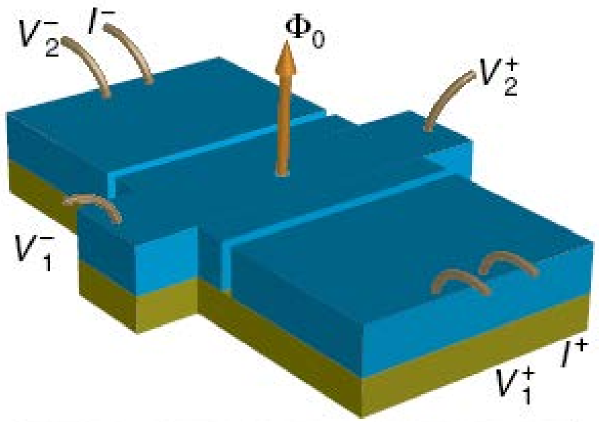


Universiteit Antwerpen

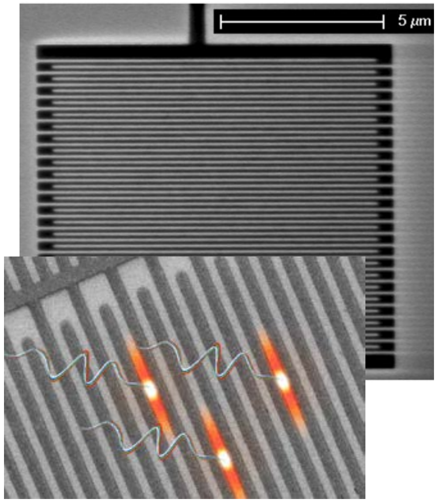
T. Golod *et al.*,
Nature Comm. 6, 8628 (2015)

Ultra-sensitive and ultra-low power superconducting electronics

AVRAM

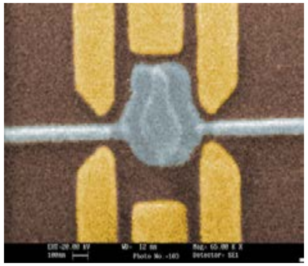


SNSPD

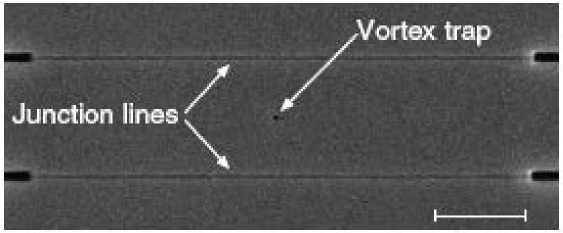


Bolometers,
THz detectors and
emitters, etc.

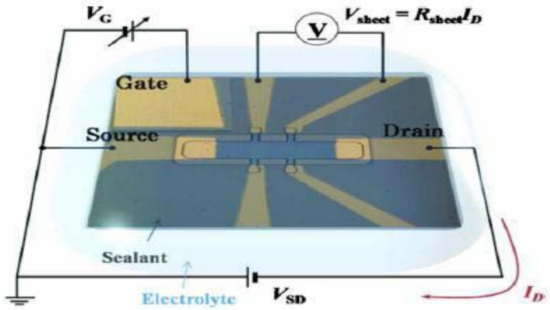
SSET

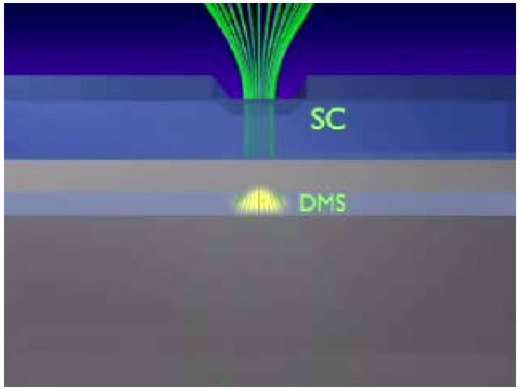


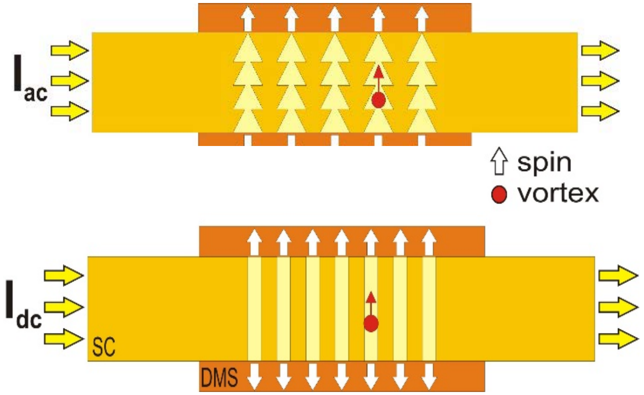
SQUID detectors



SFET



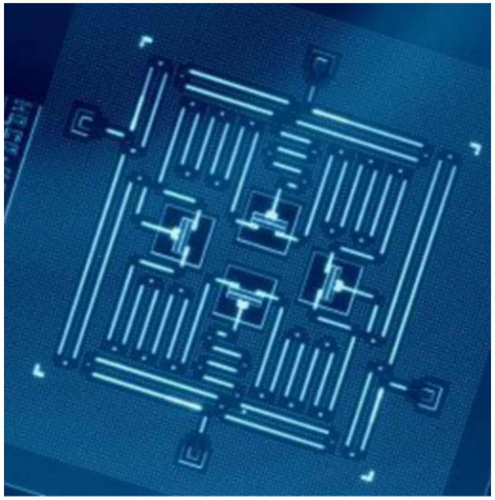


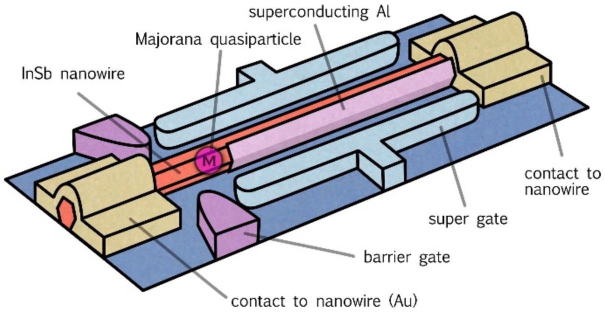


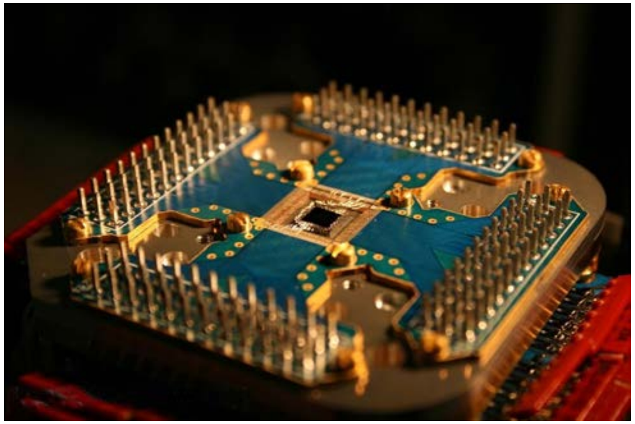
Second quantum revolution is under way!

IQM

IBM transmon Qbit | Microsoft Majorana Qbit | Google Josephson Qbit







ALL COMMERCIAL QUBIT CONCEPTS ARE BASED ON SUPERCONDUCTORS

Why? Because of being easy to fabricate, also with high chip density, while control/readout of input and output is very accessible.

Example

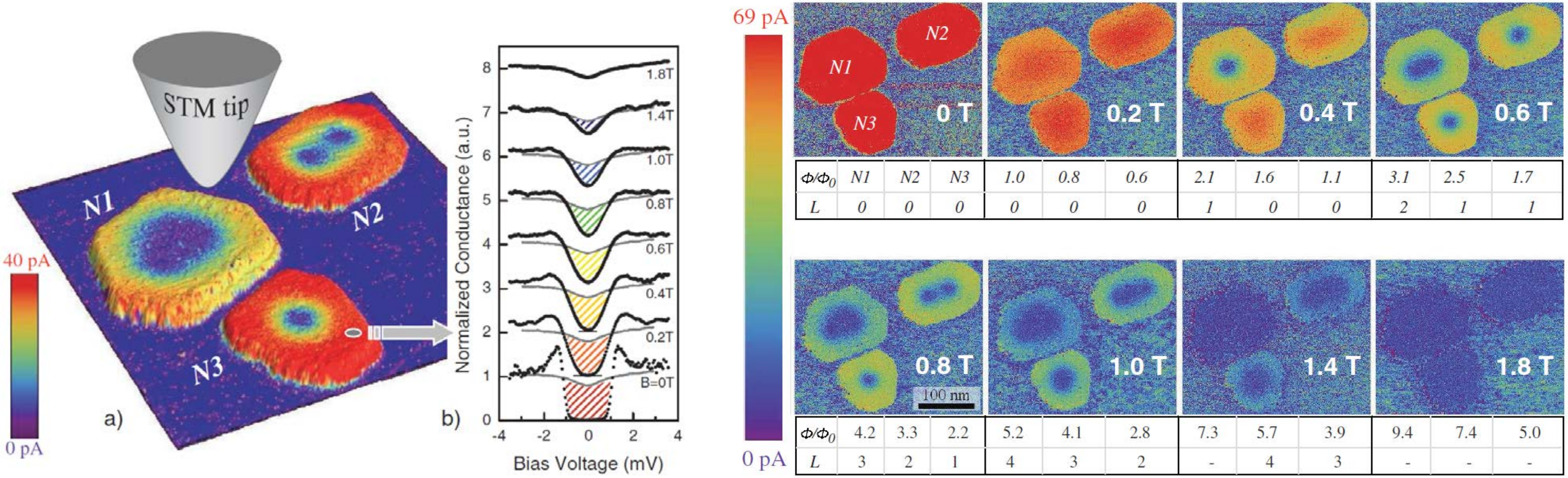
PRL **107,** 097202 (2011) PHYSICAL REVIEW LETTERS week ending 26 AUGUST 2011

Vortex Fusion and Giant Vortex States in Confined Superconducting Condensates

T. Cren, L. Serrier-Garcia, F. Debontridder, and D. Roditchev*

Institut des Nanosciences de Paris, Université Pierre et Marie Curie-Paris 6 and CNRS-UMR 7588, 4 place Jussieu, 75252 Paris, France

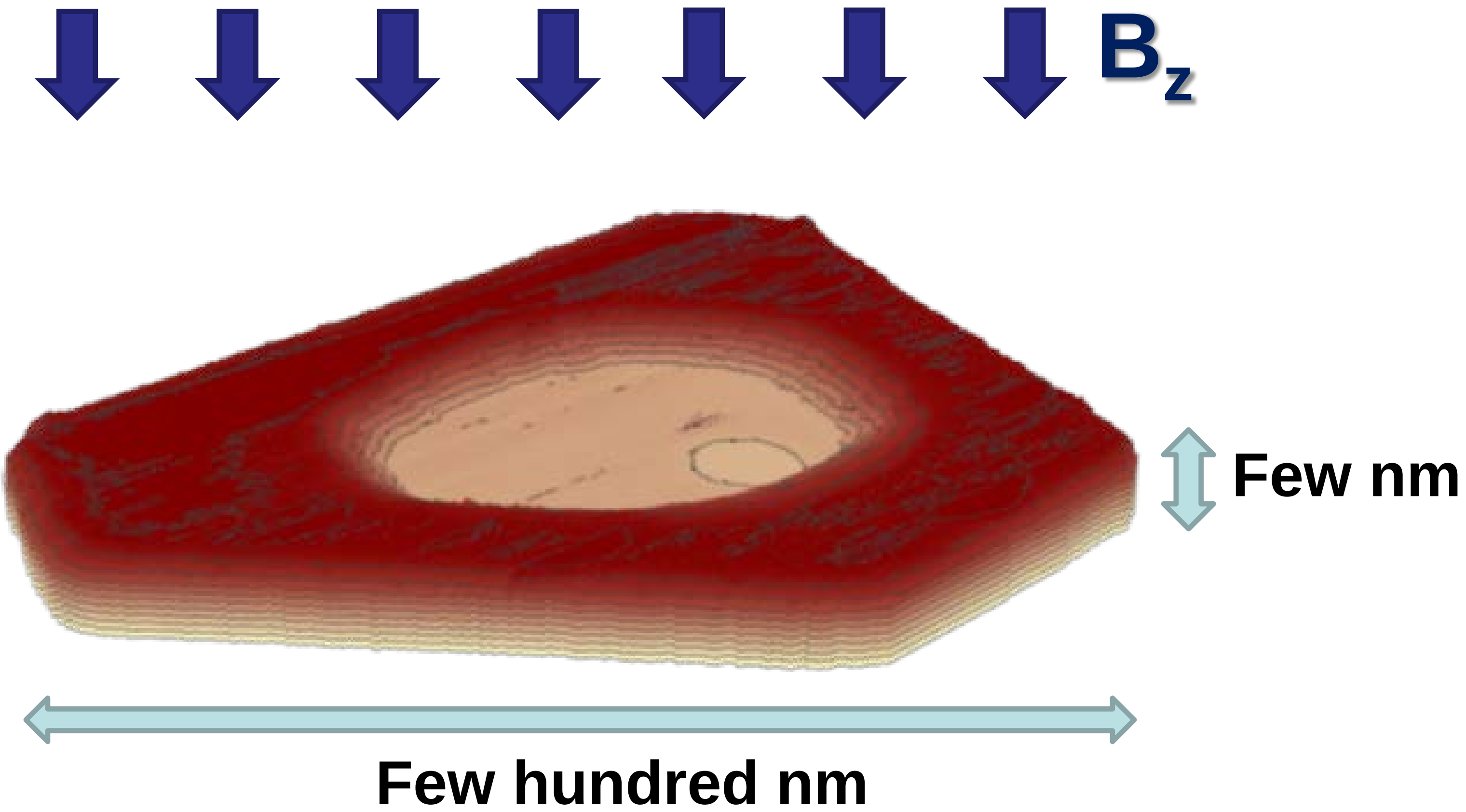
(Received 18 April 2011; published 23 August 2011)

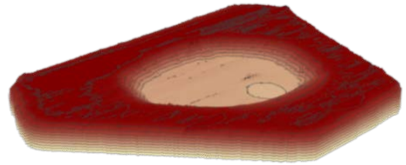


Φ/Φ_0	N1	N2	N3	1.0	0.8	0.6	2.1	1.6	1.1	3.1	2.5	1.7
L	0	0	0	0	0	0	1	0	0	2	1	1

Φ/Φ_0	4.2	3.3	2.2	5.2	4.1	2.8	7.3	5.7	3.9	9.4	7.4	5.0
L	3	2	1	4	3	2	-	4	3	-	-	-

Crystalline Pb on Si





Gibbs free energy:

$$F=\frac{H_c^2}{4\pi}\left\{\int dV\left[-(1-\frac{T}{T_c})(|\psi|^2+\frac{1}{2}|\psi|^4)+\left|\left(-i\nabla-\vec{A}\right)\psi\right|^2+\kappa^2\left(\vec{h}(\vec{r})-\vec{H}\right)^2\right]\right\}$$

All distances are expressed in coherence length units ξ(0), magnetic field in $H_{c2}(0)$, and order parameter is scaled to its value in absence of magnetic field ($\sqrt{-\frac{\alpha}{\beta}}$).

Ginzburg-Landau equations: $\left(\frac{l}{l(x,y)}-\left(\frac{l}{l(x,y)}\right)^2|\psi|^2\right)$

$$\frac{\partial\psi}{\partial t}+\left(-i\vec{\nabla}-\vec{A}\right)^2\psi=(1-t)\psi(1-|\psi|^2)+i\left(-i\vec{\nabla}-\vec{A}\right)\psi\frac{\vec{\nabla}d(x,y)}{d(x,y)}\qquad t=\frac{T}{T_c}$$

$$\frac{\partial A}{\partial t}+\kappa^2\vec{\nabla}\times\vec{\nabla}\times\vec{A}-\frac{1}{2i}\left(\psi^*\vec{\nabla}\psi-\psi\vec{\nabla}\psi^*\right)-|\psi|^2\vec{A}$$

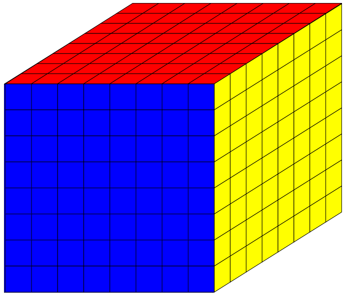
Dirty limit, $l\approx 2d\ll\xi_0$

$\xi(0)=0.855\sqrt{\xi_0\cdot l}$

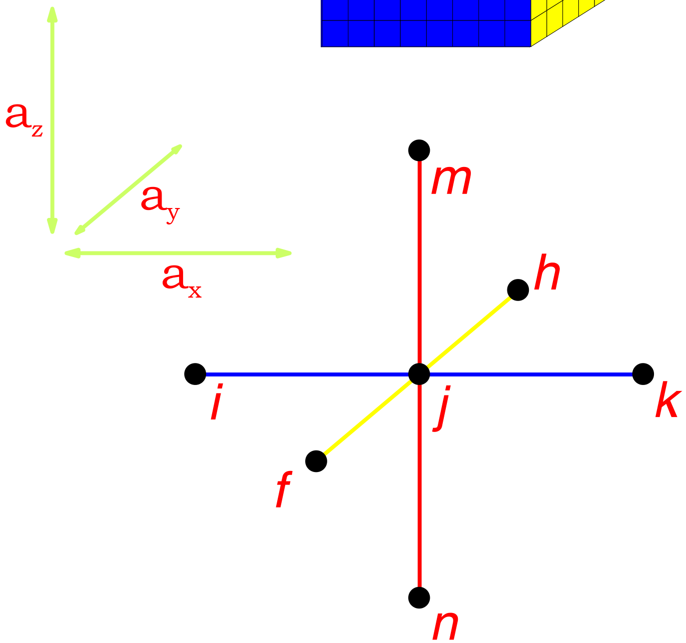
The Neumann BC is used at all SC-vacuum interfaces:

$$\left.\left(-i\hbar\vec{\nabla}-\frac{e^*}{c}\vec{A}\right)\right|_{\perp,boundary}\psi=0$$

$$\frac{\partial\psi}{\partial t} = \left(-i\vec{\nabla} - \vec{A}\right)^2 \psi - (1-t)\psi(1-|\psi|^2)$$



$$U_\mu^{\vec{r}_1,\vec{r}_2} \equiv \exp\left[-i\int_{\vec{r}_1}^{\vec{r}_2} \vec{A}_\mu(\vec{r})\cdot d\vec{\mu}\right],\ \mu = x, y, z \Rightarrow$$



$$\left(\frac{\nabla_\mu}{i} - A_\mu\right)^2 \psi_j = -\nabla_\mu^2\psi_j + i\nabla_\mu(A_\mu\psi_j) +$$

$$+A_\mu^2\psi_j + iA_\mu\nabla_\mu\psi_j = \frac{1}{U_\mu^j}\left(-2iA_\mu U_\mu^j \nabla_\mu\psi_j\right.$$

$$\left.-iU_\mu^j\psi_j(\nabla_\mu A_\mu - iA_\mu^2) + U_\mu^j\nabla_\mu^2\psi_j\right) = \frac{1}{U_\mu^j}\nabla_\mu(\nabla_\mu(U_\mu^j\psi_j))$$

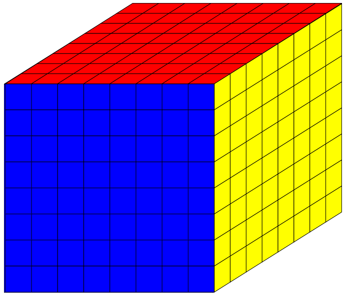
$$\left(\frac{\nabla_x}{i} - A_x\right)^2 \psi_j =$$

$$= \frac{1}{U_x^j}\frac{1}{a_x}\left(\frac{U_x^{j+1}\psi_k - U_x^j\psi_j}{a_x} - \frac{U_x^j\psi_j - U_x^{j-1}\psi_{j-1}}{a_x}\right) = \frac{U_x^{j+1,j}\psi_{j+1} - 2\psi_j + U_x^{j-1,j}\psi_{j-1}}{a_x^2}$$

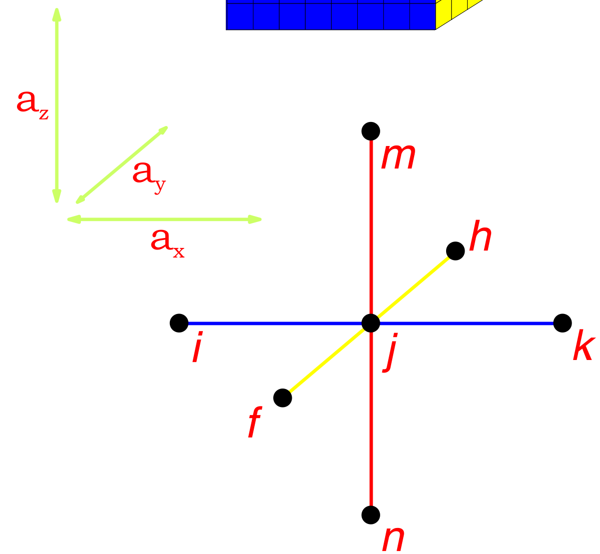
$$\eta(\psi_{jnew}-\psi_j)=$$

$$\frac{U_x^{kj}\psi_k-\psi_j}{a_x^2}+\frac{U_x^{ij}\psi_i-\psi_j}{a_x^2}+\frac{U_y^{fj}\psi_f-\psi_j}{a_y^2}+\frac{U_y^{hj}\psi_h-\psi_j}{a_y^2}$$

$$+\frac{U_z^{mj}\psi_m-\psi_j}{a_z^2}+\frac{U_z^{nj}\psi_n-\psi_j}{a_z^2}+(1-T)\left(|\psi_j|^2-1\right)\psi_j$$



Supercurrent: $\vec{j}=\frac{1}{2}\left[\psi^*\left(\frac{1}{i}\nabla-\vec{A}\right)\psi+\psi\left(\frac{1}{i}\nabla-\vec{A}\right)^*\psi^*\right]$



$$\left(\frac{1}{i}\nabla_x-A_x\right)\psi_j\rightarrow$$

$$-i\frac{1}{U_x^j}\nabla_x(U_x^j\psi_j)=-i\frac{U_x^{kj}\psi_k-\psi_j}{a_x}$$ **=0**, at the boundary

Notice that link variables take simple form:

$$U_x^{k,j}=\exp\left[-i\int_k^j A_x(x)\cdot dx\right]=\exp(-i\frac{1}{2}B_z y a_x)$$

Simulation

Generalized time-dependent Ginzburg-Landau framework

First gTDGL equation:

$$\frac{\pi\hbar}{8k_BT_cu}\frac{u}{\sqrt{1+(2\tau_i|\Delta|\hbar^{-1})^2}}\left[\frac{\partial}{\partial t}+i\frac{e^*}{\hbar}\varphi+\frac{1}{2}\frac{\partial}{\partial t}(2\tau_i|\Delta|\hbar^{-1})^2\right]\Delta=$$

$$=\frac{\pi\hbar D}{8k_BT_c}\left(\nabla-i\frac{e^*}{\hbar}\boldsymbol{A}\right)^2\Delta+\left[f(T)-g(T)\frac{\pi^2}{16uk_B^2T_c^2}|\Delta|^2\right]\Delta$$

$$f(T)=\frac{1-\left(\frac{T}{T_c}\right)^2}{1+\left(\frac{T}{T_c}\right)^2}$$

$$g(T)=\left[1+\left(\frac{T}{T_c}\right)^2\right]^{-2}$$

Second gTDGL equation:

$$\frac{1}{\mu_0}\nabla\times\nabla\times\boldsymbol{A}=\sigma_n\left[\frac{\pi}{2k_BT_ce^*}|\Delta|^2\left(\nabla\theta-\frac{e^*}{\hbar}\boldsymbol{A}\right)-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\varphi\right]+\nu_{el}^{(ext)}$$

Current conservation law:

$$\nabla\left[\sigma_n\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi\right)\right]=\nabla\left[\frac{\sigma_n\pi}{2k_BT_ce^*}|\Delta|^2\left(\nabla\theta-\frac{e^*}{\hbar}\boldsymbol{A}\right)\right]+\nabla\nu_{el}^{(ext)}$$

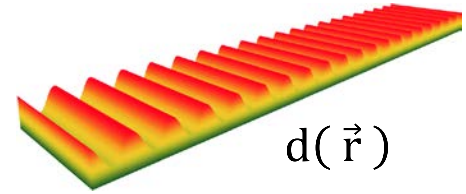
Equation of thermal balance:

$$C\frac{\partial T}{\partial t}=K\nabla^2T-\frac{h}{d}(T-T_0)+\sigma_n\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi\right)^2+\nu_{th}^{(ext)}$$

GLACE

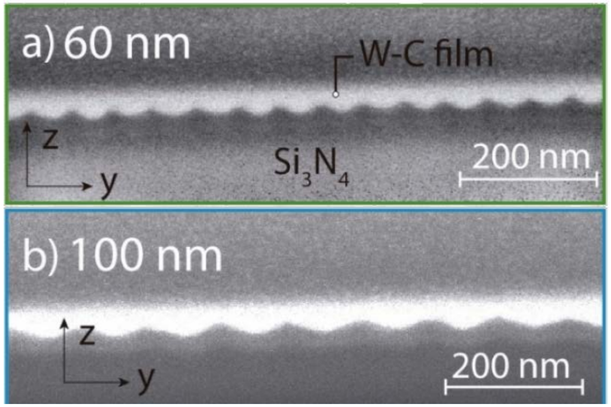
Generalized time-dependent Ginzburg-Landau framework

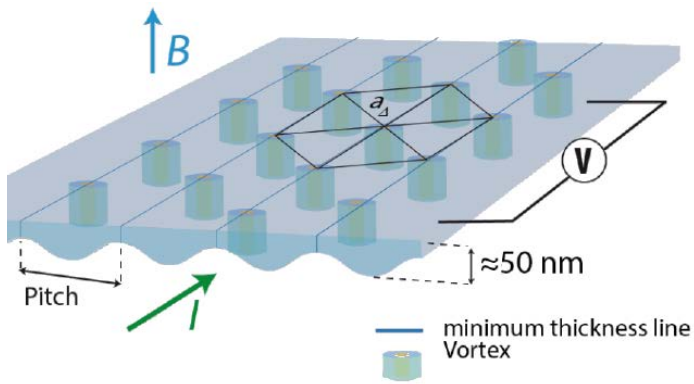
Variable thickness

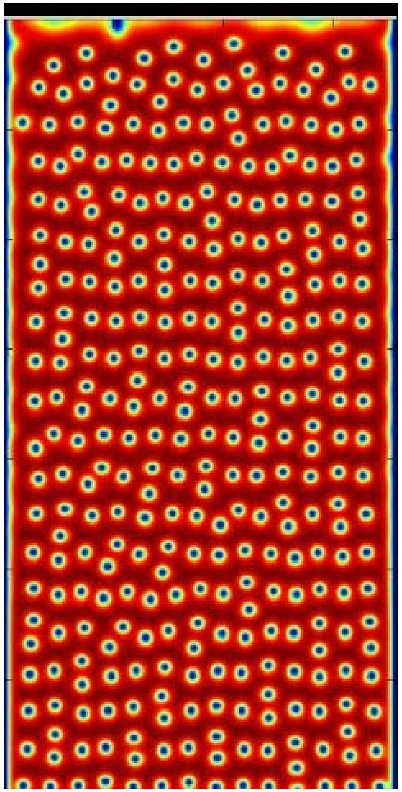


$d(\vec{r})$

Collaboration with groups of J. M. De Teresa and H. Suderow







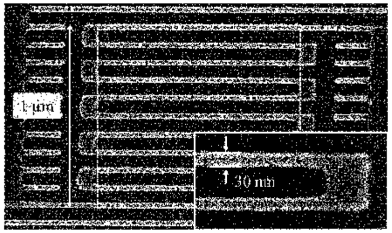
$$\frac{\pi\hbar}{8k_BT_cu}\frac{u}{\sqrt{1+(2\tau_i|\Delta|\hbar^{-1})^2}}\left[\frac{\partial}{\partial t}+i\frac{e^*}{\hbar}\varphi+\frac{1}{2}\frac{\partial}{\partial t}(2\tau_i|\Delta|\hbar^{-1})^2\right]\Delta =$$

$$= \frac{\pi\hbar D}{8k_BT_c}\left(\nabla - i\frac{e^*}{\hbar}\boldsymbol{A}\right)^2\Delta + \left[f(T)-g(T)\frac{\pi^2}{16uk_B^2T_c^2}|\Delta|^2\right]\Delta +$$

$$+\frac{\pi\hbar D}{8k_BT_c}\frac{\nabla d}{d}\left(\nabla - i\frac{e^*}{\hbar}\boldsymbol{A}\right)\Delta$$

I. Serrano *et al.*, *Beilstein* J. Nanotechnol. ***7***, 1698 (2016)
I. Guillamón *et al.*, Nature Physics **10**, 851 (2014)

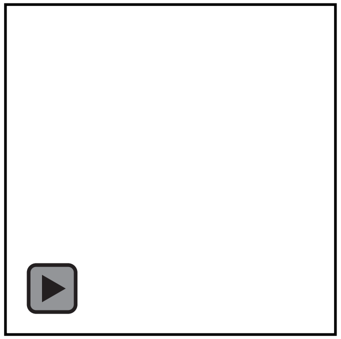
What other experimentally relevant parameters can be taken into account?

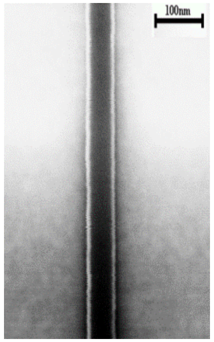


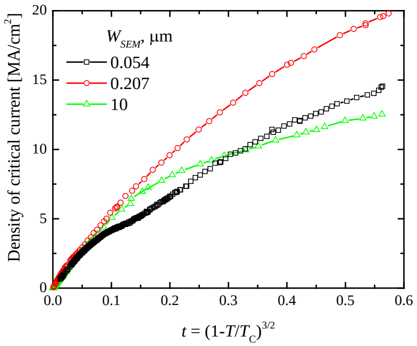
Courtesy of K. Il'in, KIT

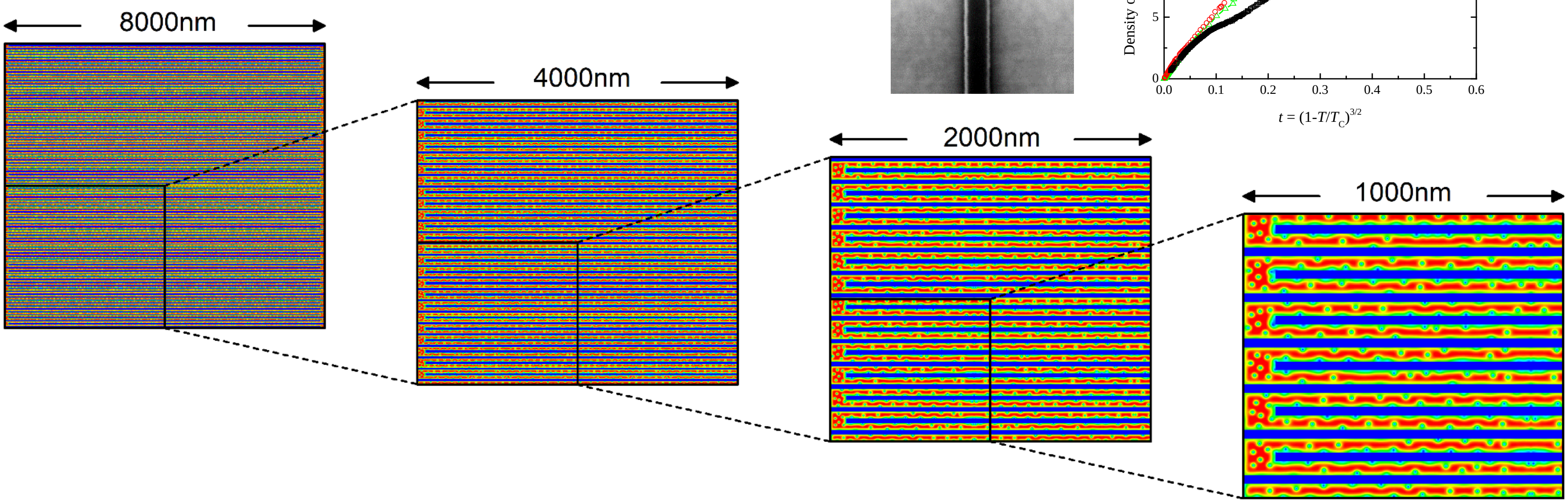
- Arbitrary geometry and placement of leads

Corbino geometry



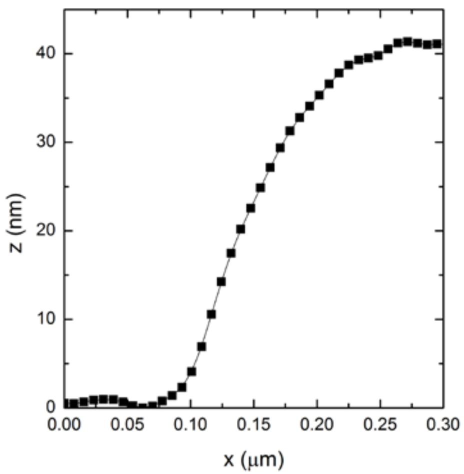


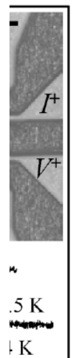




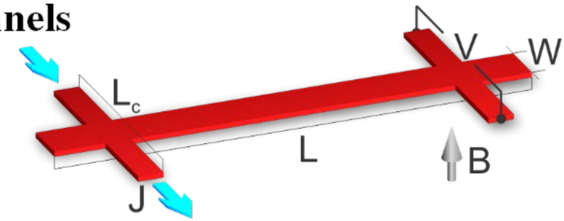
Size matters

Long-Range Nonlocal Flow of Vortices in Narrow Superconducting Channels





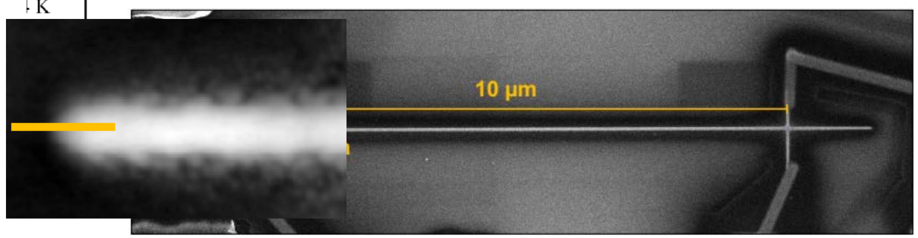
Grigorieva et al. PRL ***92***, 237001 (2004)

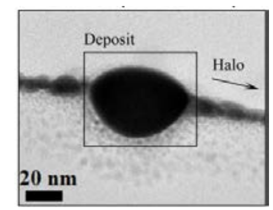


But:

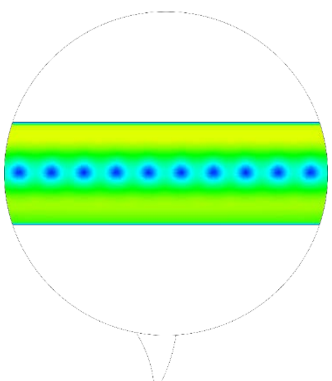
- Effect weak
- Realized over maximum one micron distance

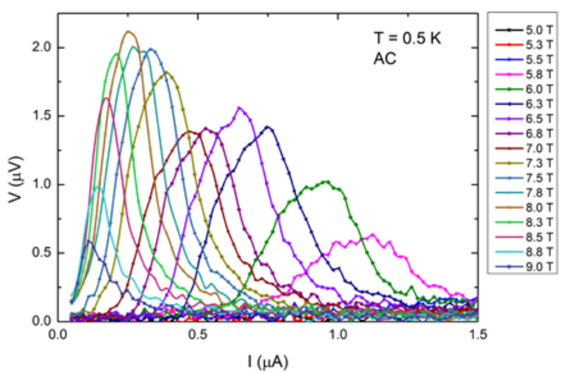
We recognized the design fault, and revisited the problem in simulations and experiment:

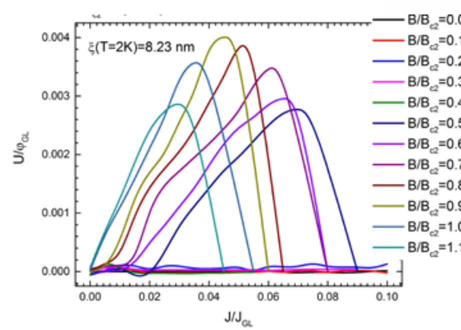


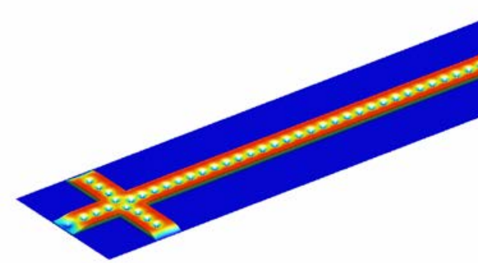


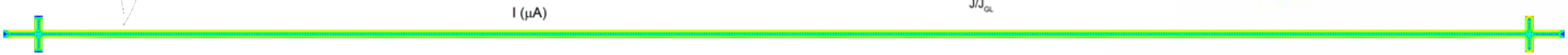
Collaboration De Teresa @UZaragoza and Suderow @UAM

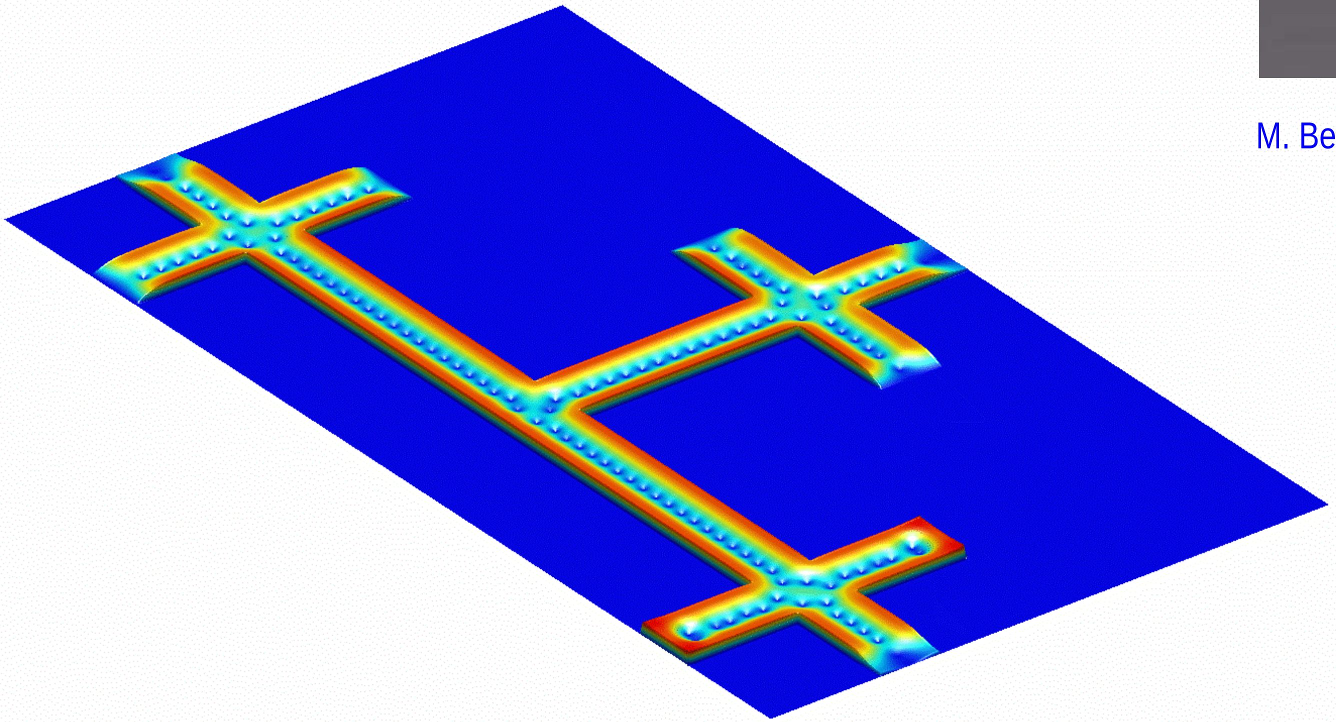


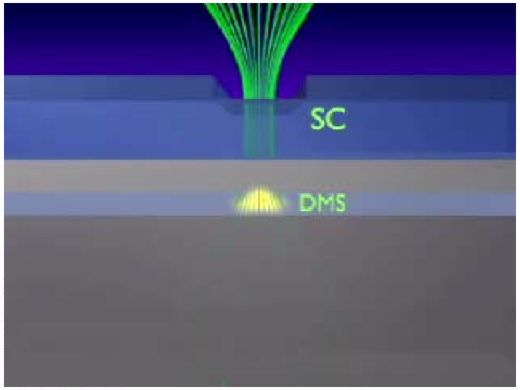












M. Berciu *et al.*, Nature **435**, 71 (2005)

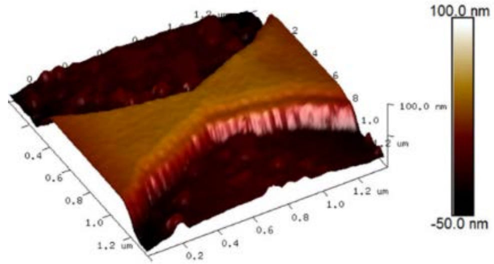
What other experimentally relevant parameters can be taken into account?

Material-dependent or sample-specific parameters

$T_c(\boldsymbol{r}),\ \tau_i(\boldsymbol{r}),\ D(\boldsymbol{r}) = D(\ell(\boldsymbol{r}))$ $\qquad d(\boldsymbol{r})$

$\sigma_n(\boldsymbol{r})$

$C(\boldsymbol{r}),\ K(\boldsymbol{r}),\ h(\boldsymbol{r})$

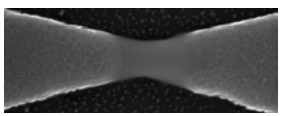


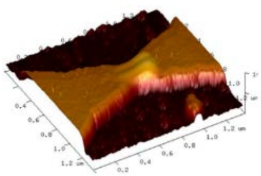
Collaboration Silhanek @ULiege, Van de Vondel @KULeuven

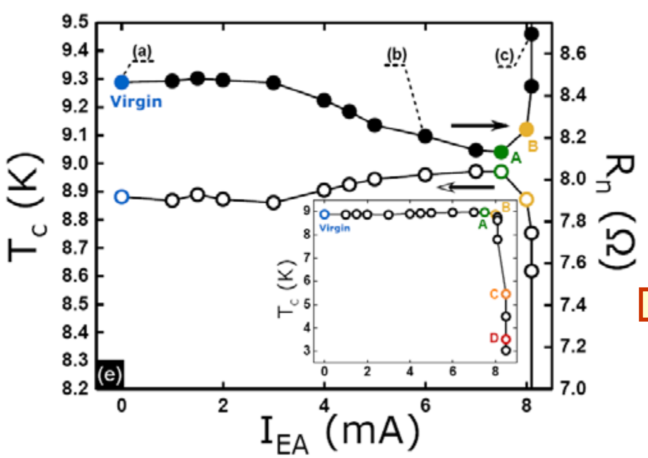
Baumans *et al.*,
Nat. Comm. **7**, 10560 (2017)

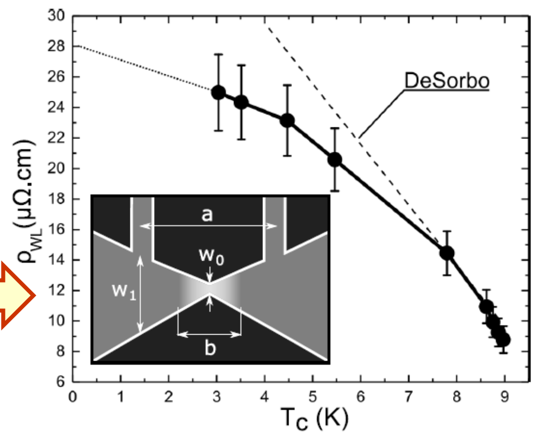
In situ tailoring of superconducting junctions via electro-annealing†

Nanoscale **10, 1987 (2018)**









$$R_N = \frac{a}{d(w_1 - w_0)}\left[\rho_{WL}\ln\left(1 + \frac{(w_1 - w_0)b}{w_0 a}\right) - \rho_0 \ln\left(\frac{w_0}{w_1} + \frac{(w_1 - w_0)b}{w_0 a}\right)\right]$$

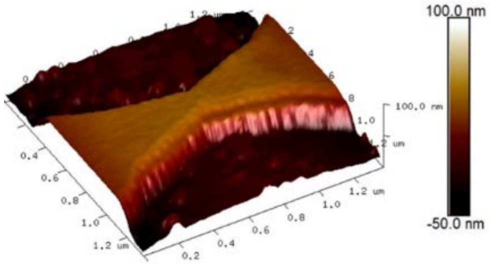
What other experimentally relevant parameters can be taken into account?

Material-dependent or sample-specific parameters

$T_c(\mathbf{r}),\ \tau_i(\mathbf{r}),\ D(\mathbf{r}) = D(\ell(\mathbf{r}))$ $\quad d(\mathbf{r})$

$\sigma_n(\mathbf{r})$

$C(\mathbf{r}),\ K(\mathbf{r}),\ h(\mathbf{r})$



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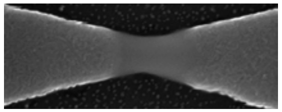
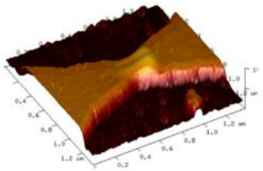
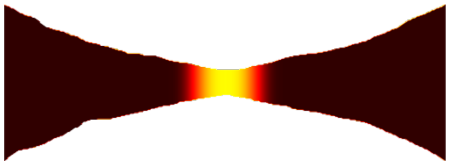
Baumans *et al.*,
Nat. Comm. **7**, 10560 (2017)

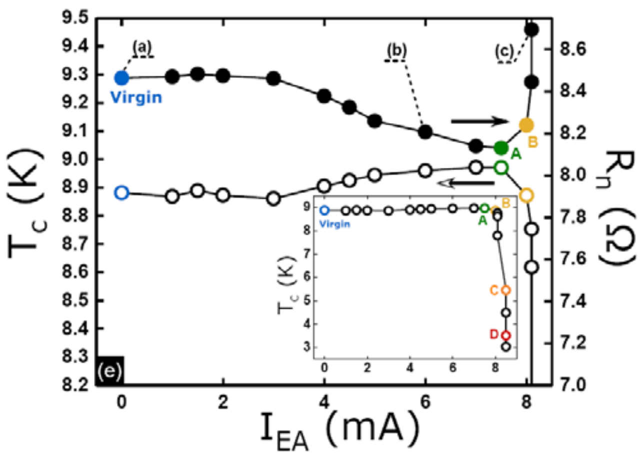
In situ tailoring of superconducting junctions via electro-annealing†

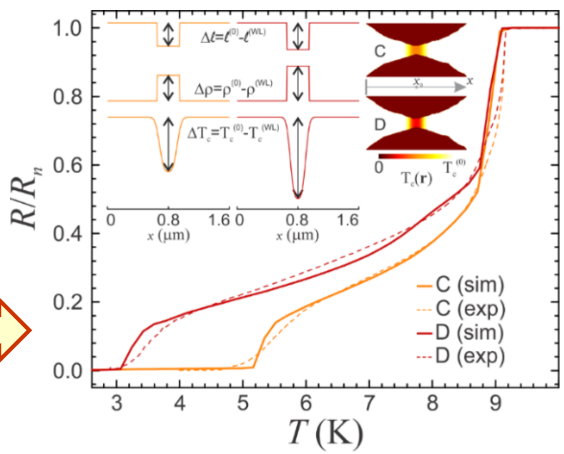
***Nanoscale* 10, 1987 (2018)**

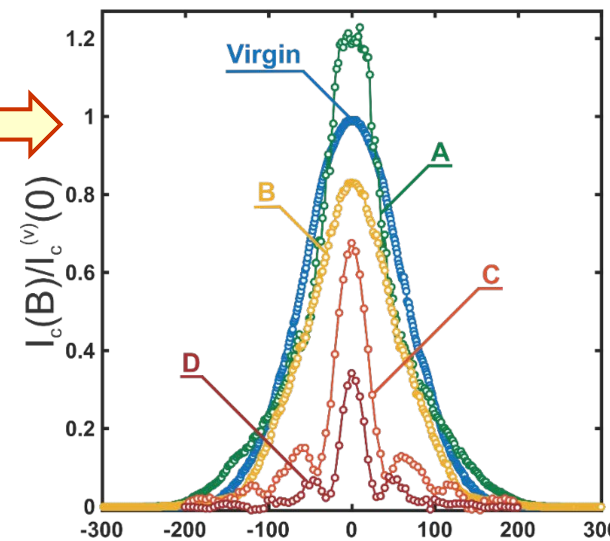












Generalized time-dependent Ginzburg-Landau framework

First gTDGL equation:

$$\frac{\pi\hbar}{8k_BT_cu}\frac{u}{\sqrt{1+(2\tau_i|\Delta|\hbar^{-1})^2}}\left[\frac{\partial}{\partial t}+i\frac{e^*}{\hbar}\varphi+\frac{1}{2}\frac{\partial}{\partial t}(2\tau_i|\Delta|\hbar^{-1})^2\right]\Delta=$$

$$=\frac{\pi\hbar D}{8k_BT_c}\left(\nabla-i\frac{e^*}{\hbar}\boldsymbol{A}\right)^2\Delta+\left[f(T)-g(T)\frac{\pi^2}{16uk_B^2T_c^2}|\Delta|^2\right]\Delta$$

$$f(T)=\frac{1-\left(\frac{T}{T_c}\right)^2}{1+\left(\frac{T}{T_c}\right)^2}$$

$$g(T)=\left[1+\left(\frac{T}{T_c}\right)^2\right]^{-2}$$

Second gTDGL equation:

$$\frac{1}{\mu_0}\nabla\times\nabla\times\boldsymbol{A}=\sigma_n\left[\frac{\pi}{2k_BT_ce^*}|\Delta|^2\left(\nabla\theta-\frac{e^*}{\hbar}\boldsymbol{A}\right)-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\varphi\right]+\nu_{el}^{(ext)}$$

Current conservation law:

$$\nabla\left[\sigma_n\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi\right)\right]=\nabla\left[\frac{\sigma_n\pi}{2k_BT_ce^*}|\Delta|^2\left(\nabla\theta-\frac{e^*}{\hbar}\boldsymbol{A}\right)\right]+\nabla\nu_{el}^{(ext)}$$

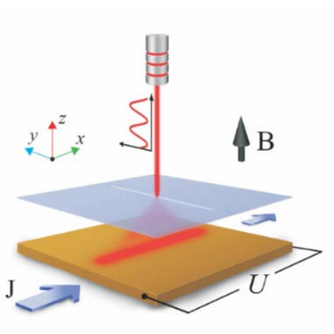
Equation of thermal balance:

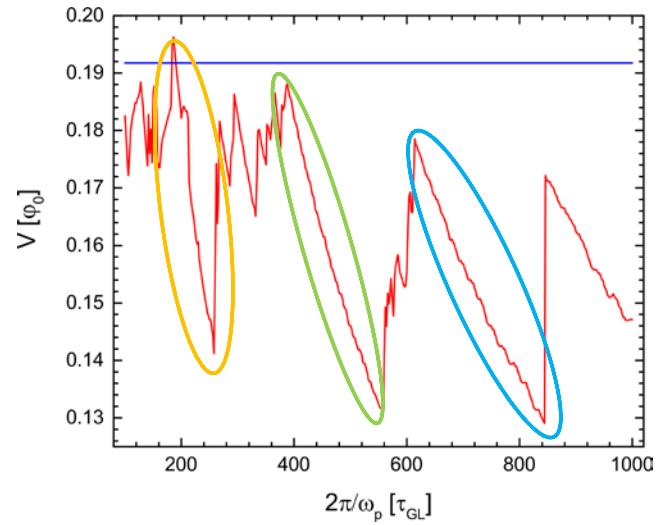
$$C\frac{\partial T}{\partial t}=K\nabla^2T-\frac{h}{d}(T-T_0)+\sigma_n\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi\right)^2+\nu_{th}^{(ext)}$$

What other experimentally relevant parameters can be taken into account?

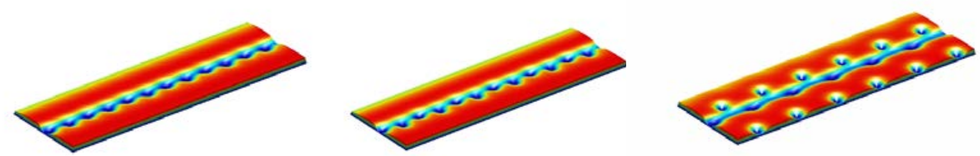
- External potentials?
  - in order to simulate effects of pulsed excitations

$\nu_{el}^{(ext)}$, $\nu_{th}^{(ext)}$





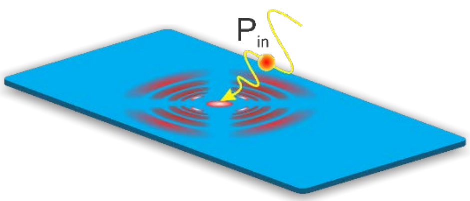
Jelic *et al.* Sci. Rep. **5**, 14604 (2015)
Jelic *et al.* Sci. Rep. **6**, 35687 (2016)

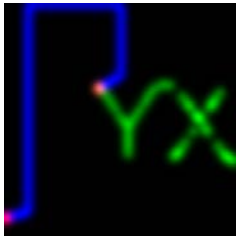


What other experimentally relevant parameters can be taken into account?

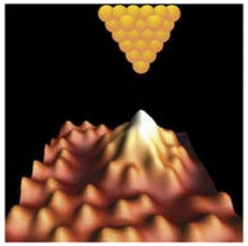
- External potentials?
  - in order to simulate effects of pulsed excitations
  - In order to simulate effects of LTSLM, STM, SNSPD…

$\nu_{el}^{(ext)}$, $\nu_{th}^{(ext)}$

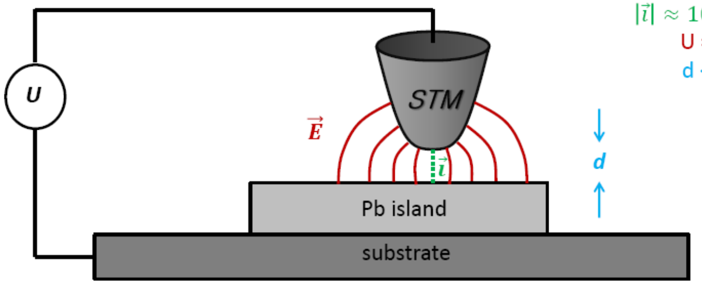




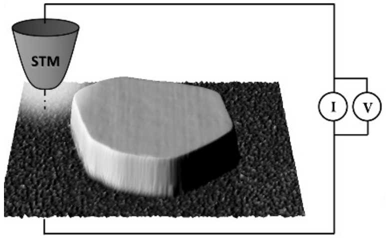
I. Veshchunov *et al.*,
Nat. Comm. 7, 12801 (2016)

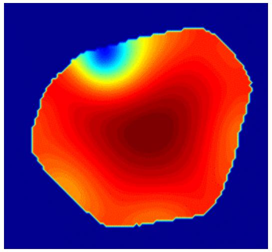


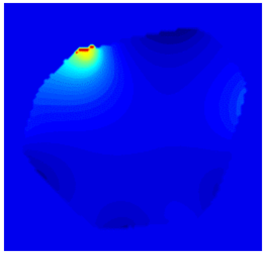
Junyi Ge *et al.*,
Nat. Comm. 7, 13880 (2016)



T. Cren *et al.*,
Phys. Rev. Lett. 107, 097202 (2011)

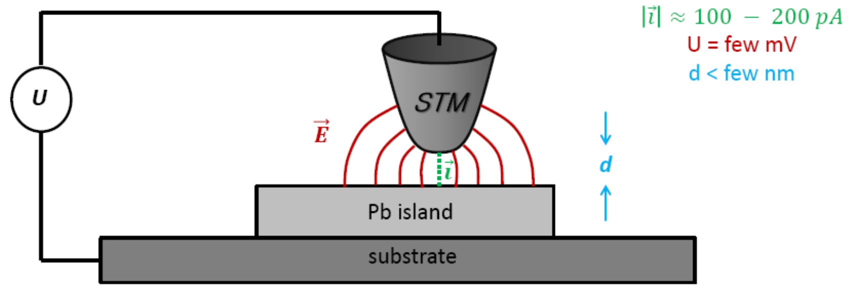




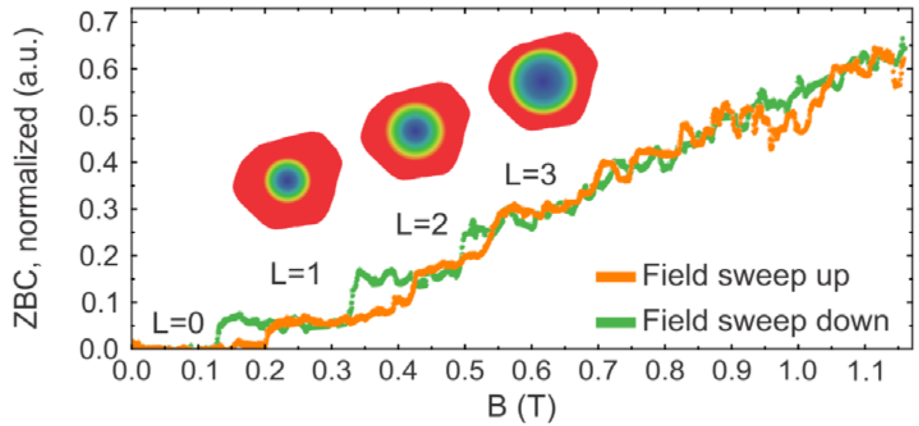


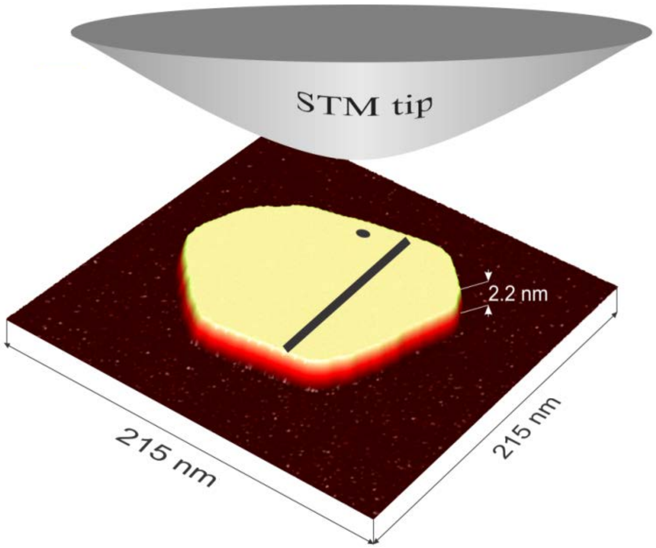
Gating by an STM tip

Collaboration Cren, Roditchev, Brun @INSP, UPMC

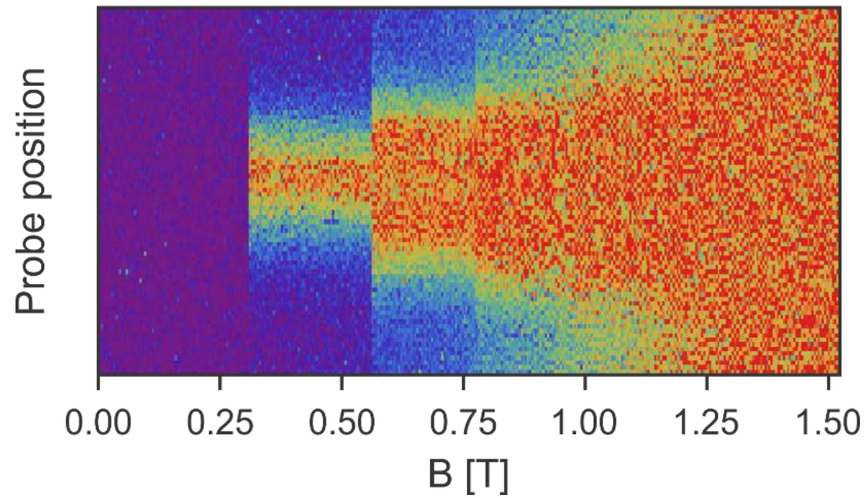


Cren et al., Phys. Rev. Lett. 107 (2011)



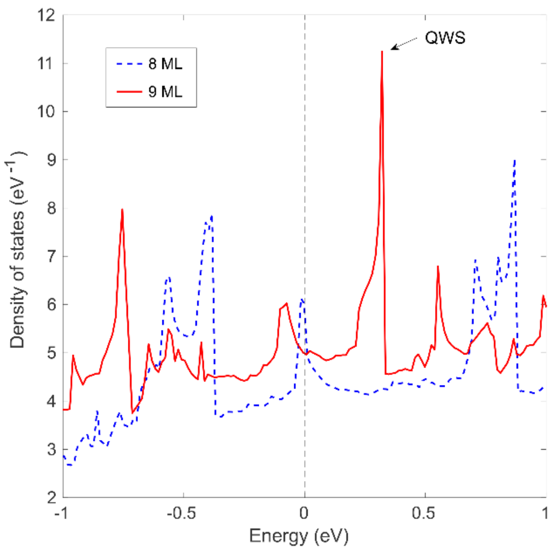


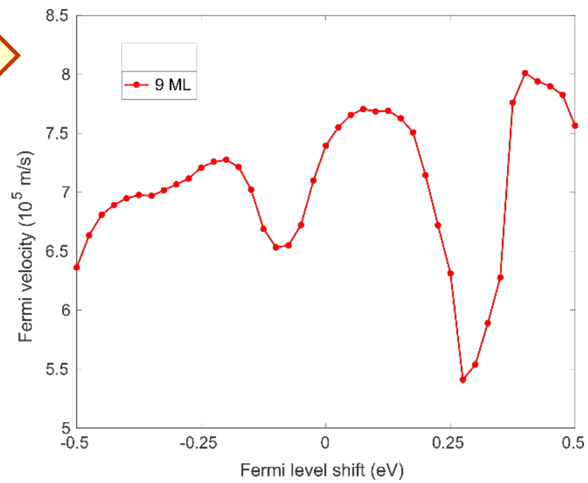
8 ML Pb island



Gating by an STM tip

9 ML Pb





Work function difference, albeit screened, sufficient to locally change coherence length by 30%!