Mesoscopic superconductivity: From Andreev transport to Andreev qubits

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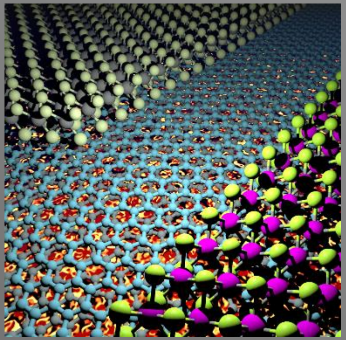
OCHOA



European School on Superconductivity and Magnetism in Quantum Materials. Valencia 21-25th of April, 2024

Outline:

Introduction to mesoscopic superconductivity

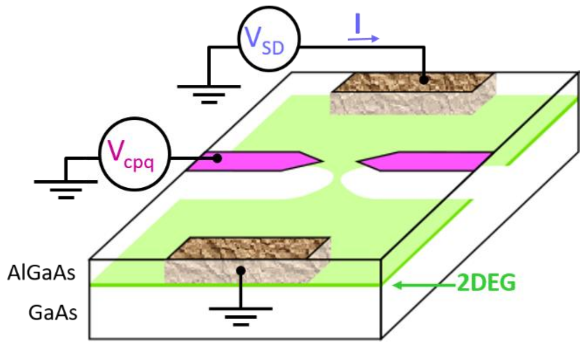
1st part: Theoretical modeling of transport

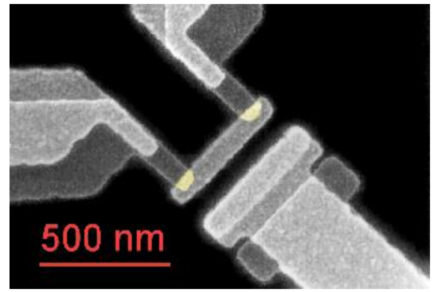
- Scattering vs Hamiltonian approach
- Superconducting atomic contacts
- Transport in Majorana nanowires

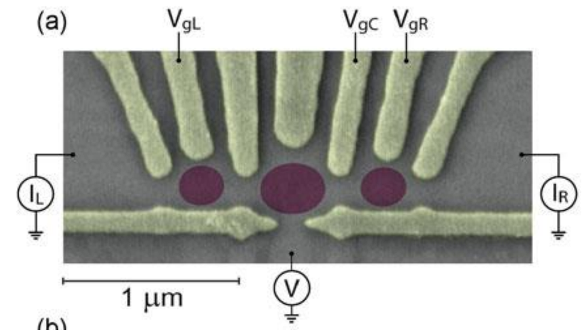
2nd part: Detection and manipulation of ABS

- Effects of length, spin-orbit and interactions
- Comparison to experiments
- Towards ABS + cQED theory
- The Andreev spin qubit

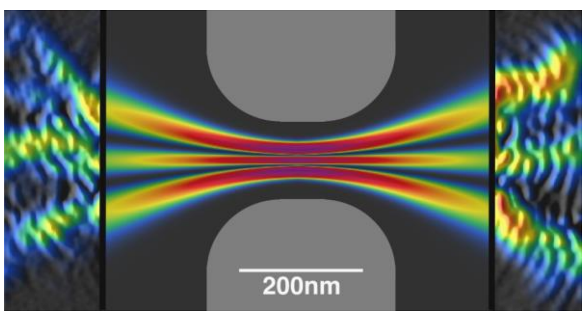
reduced dimensionality (nanoscale devices)





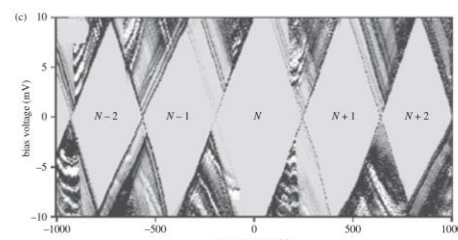


quantum transport



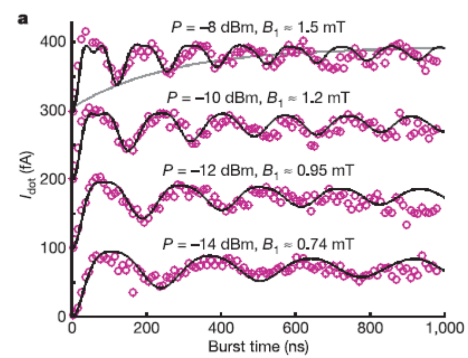
(Conductance quantization, AB effect, Quantum noise, etc)

interactions

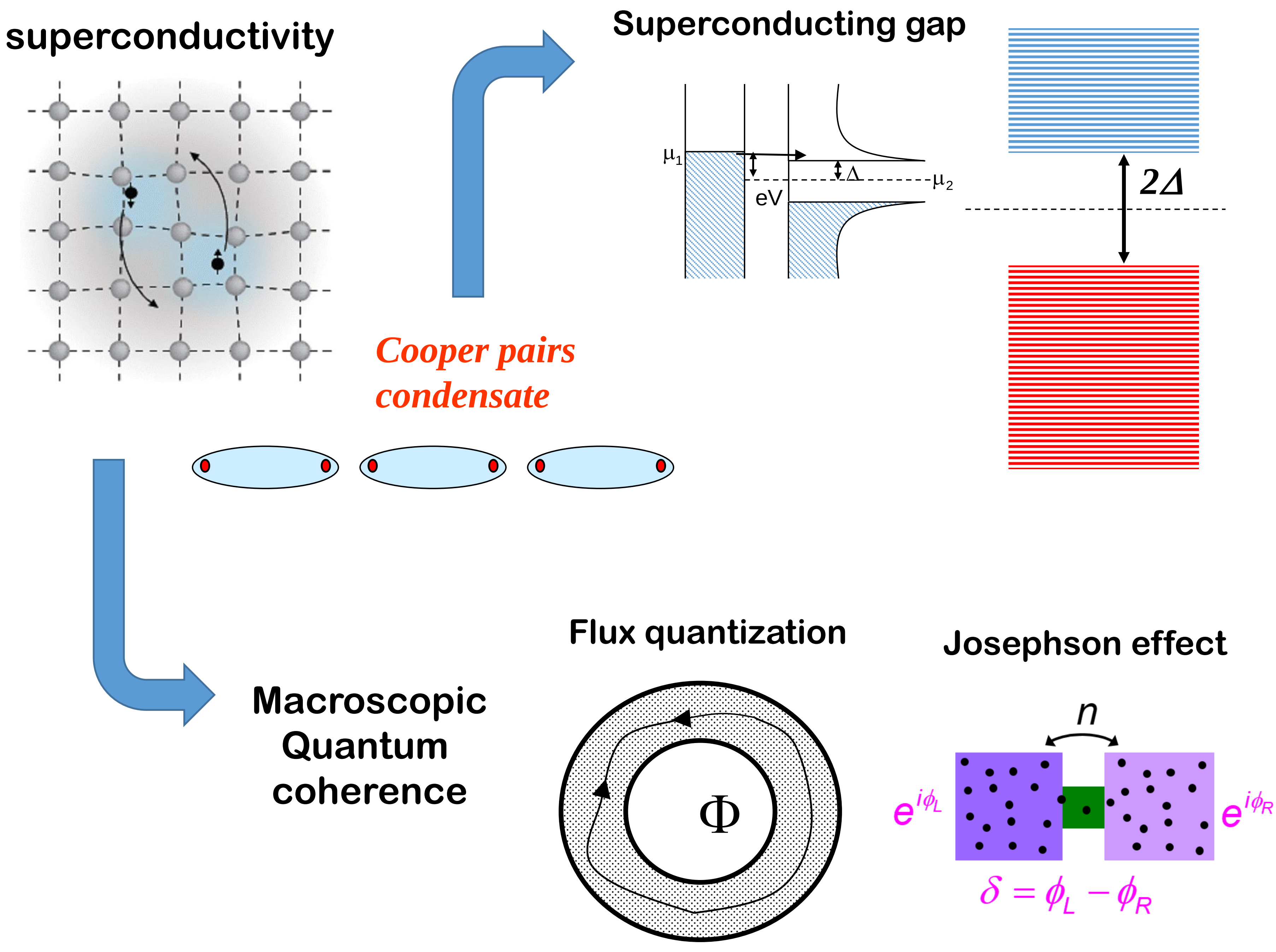


(CB, DCB, Kondo, etc)

coherent dynamics

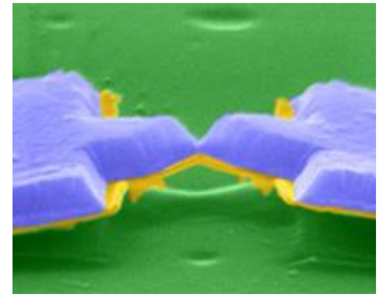


(charge and spin qubits, etc)

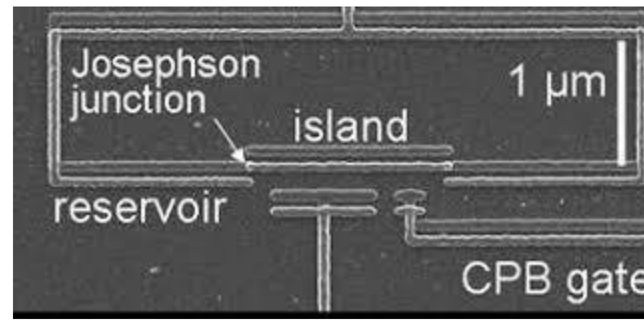

superconductivity
Superconducting gap
μ1
eV
Δ
μ2
2Δ
Cooper pairs
condensate
Macroscopic
Quantum
coherence
Flux quantization
Φ
Josephson effect
n
$e^{i\phi_L}$
$e^{i\phi_R}$
$\delta = \phi_L - \phi_R$

superconductivity at nanoscale

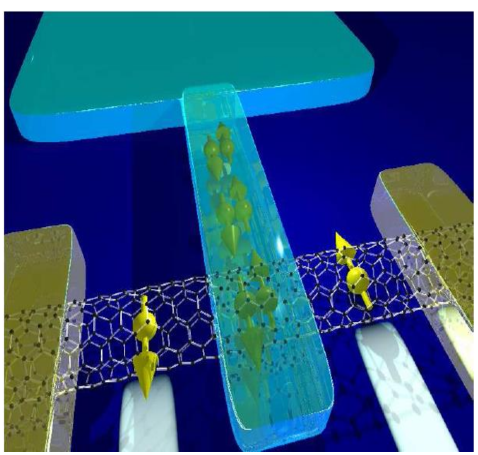
Superconducting atomic contacts



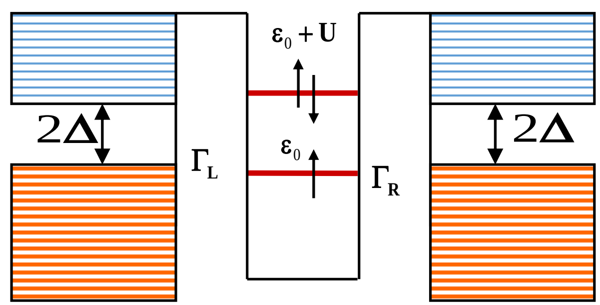
Cooper pair box

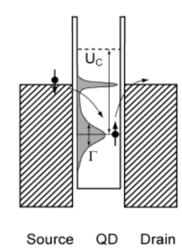


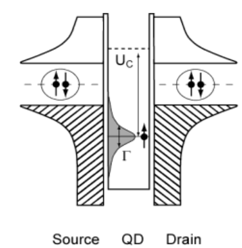
Cooper pair splitter



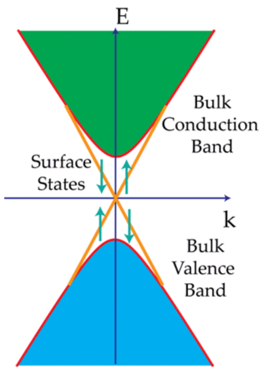
Superconducting QDs

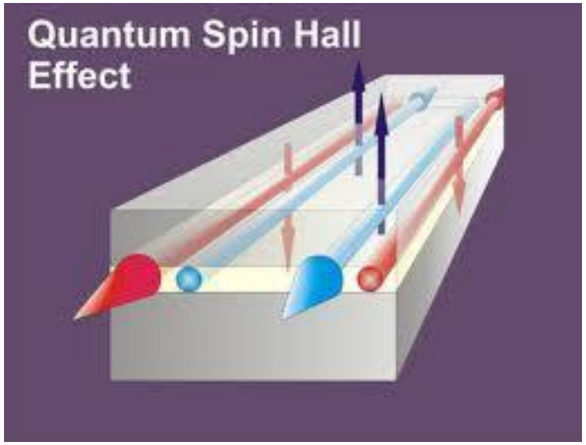


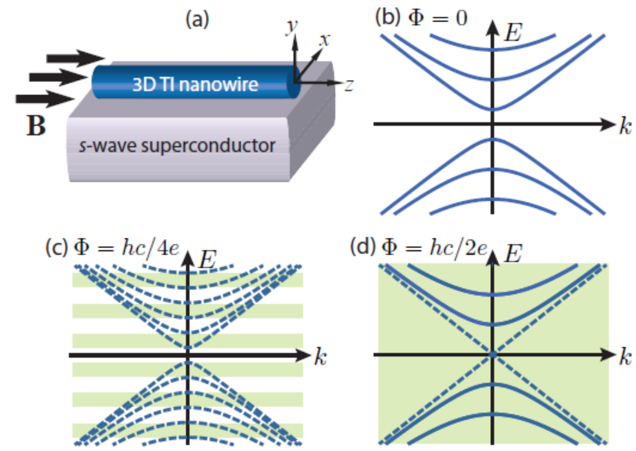




Topology

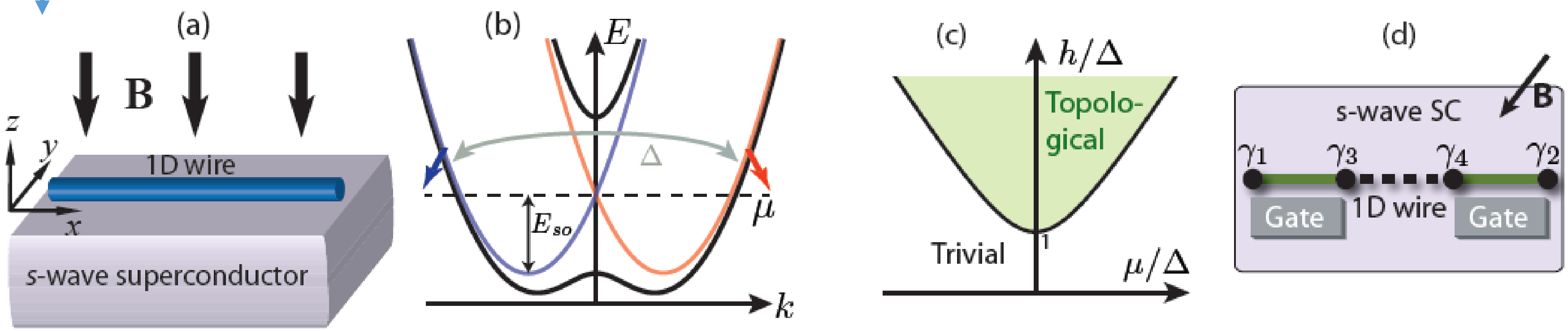






Cook et al. 2011

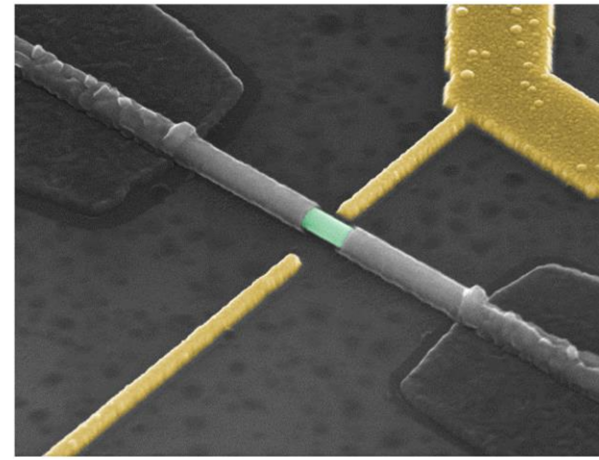
Inducing Topological Superconductivity in nanowires



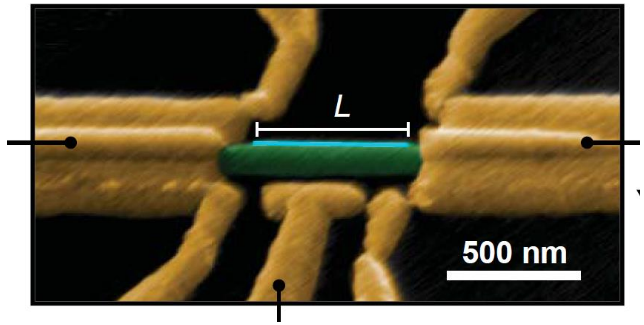
Lutchyn et al. 2010; Oreg et al. 2010

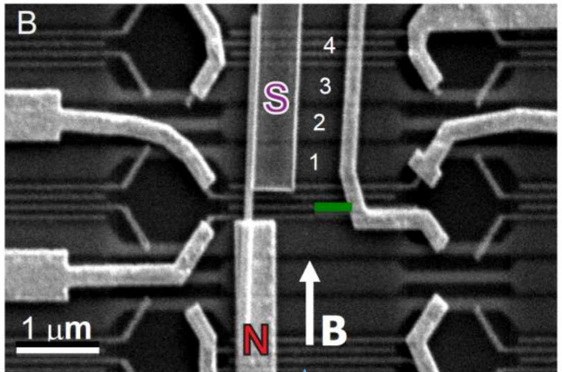
Hybrid nanowire devices

Saclay

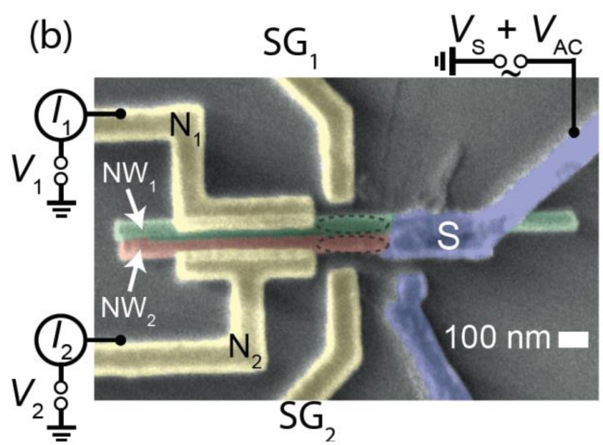


Copenhagen



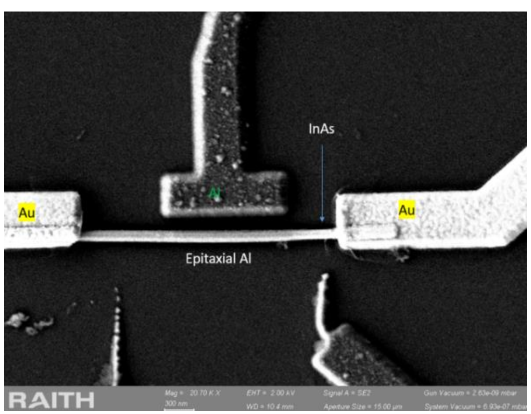


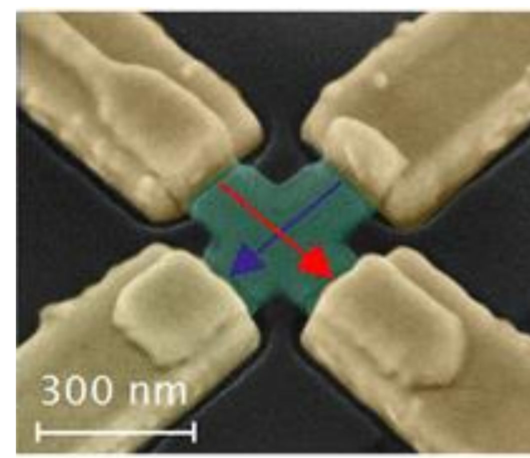
Delft



Zurich

Madrid

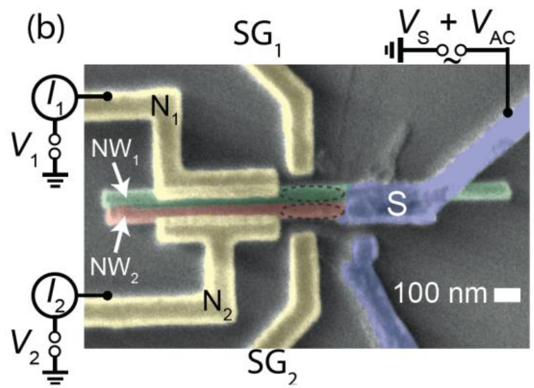


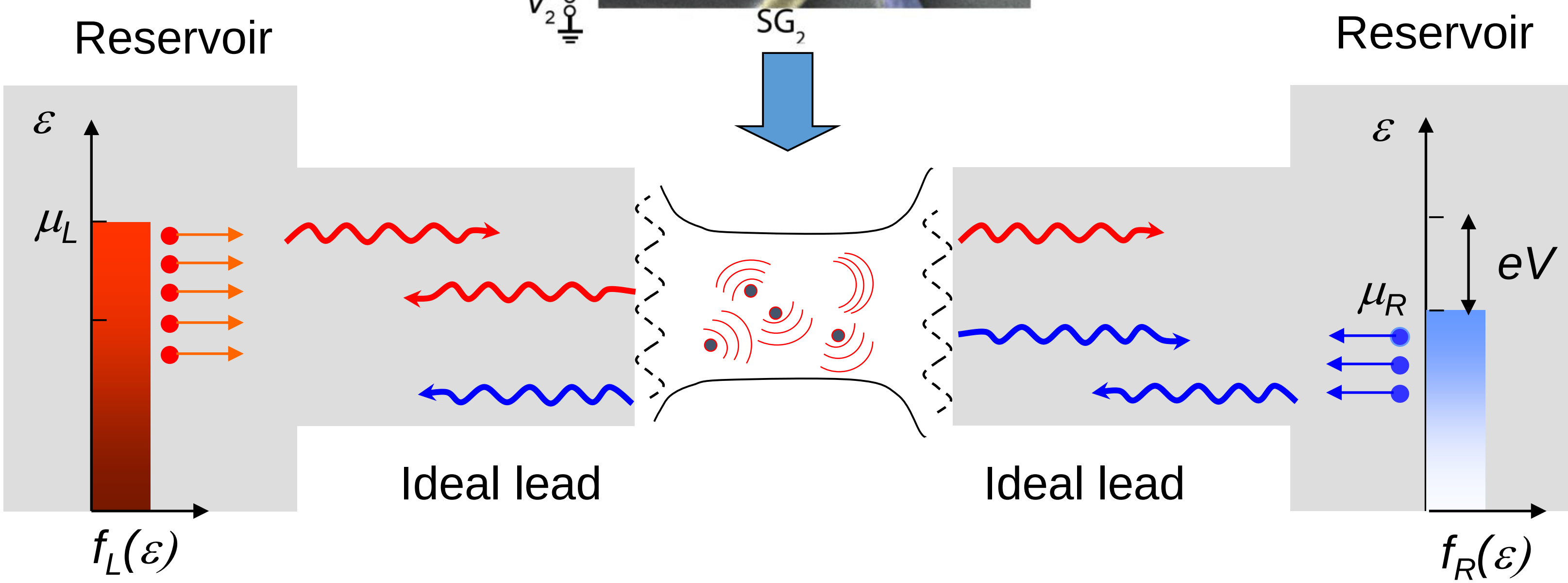


First part: theoretical modeling of transport phenomena

Basic approach to quantum transport: Landauer picture

actual system





$eV \rightarrow 0$

$$G = \frac{2e^2}{h} T(E_F)$$

$$T = \sum_n \tau_n$$

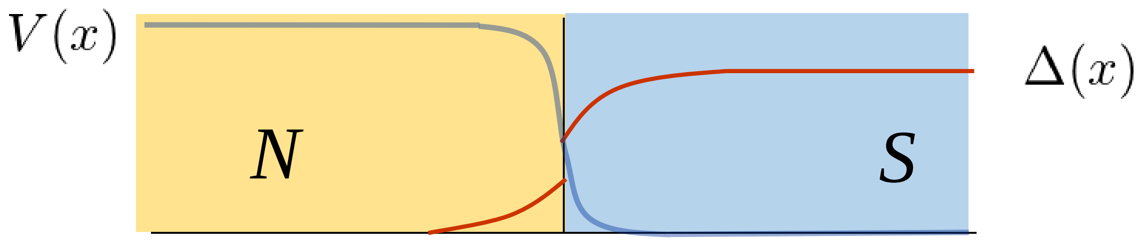
Conduction channels

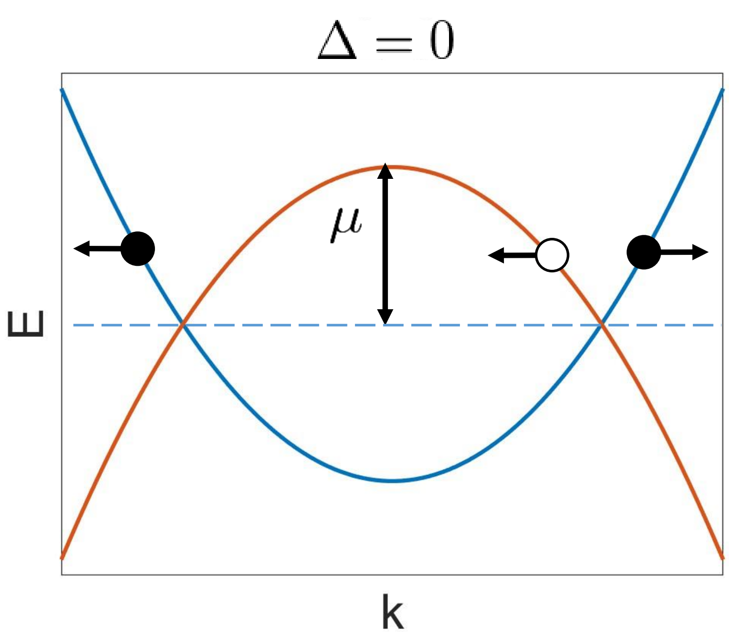
Extension to SC case: BdG description

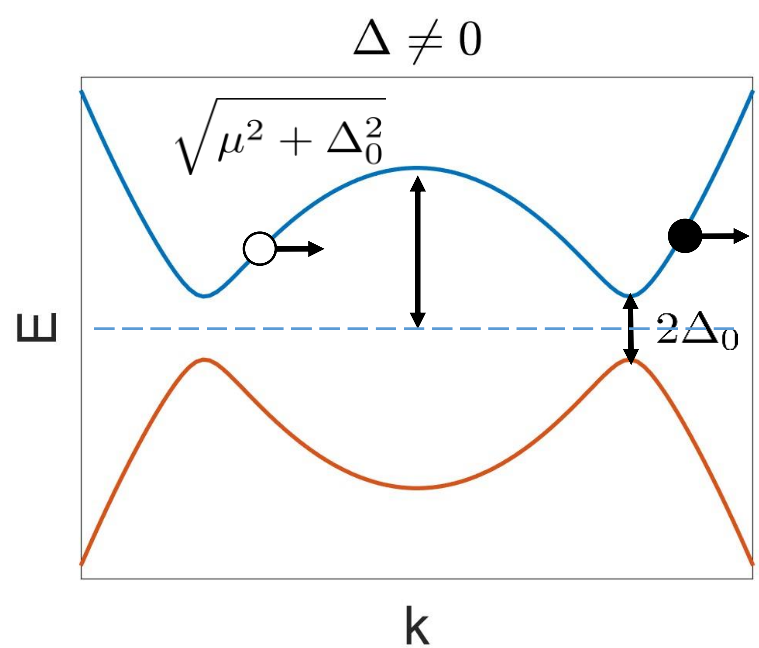
$$\begin{pmatrix} H_e(x) - \mu & \Delta(x) \\ \Delta^*(x) & \mathcal{T}^{-1}(\mu - H_e(x))\mathcal{T} \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix} = E \begin{pmatrix} u(x) \\ v(x) \end{pmatrix}$$

$\mathcal{T} \equiv$ time reversal

$$H_e(x) = -\frac{\hbar^2}{2m^*}\partial_x^2 + V(x)$$

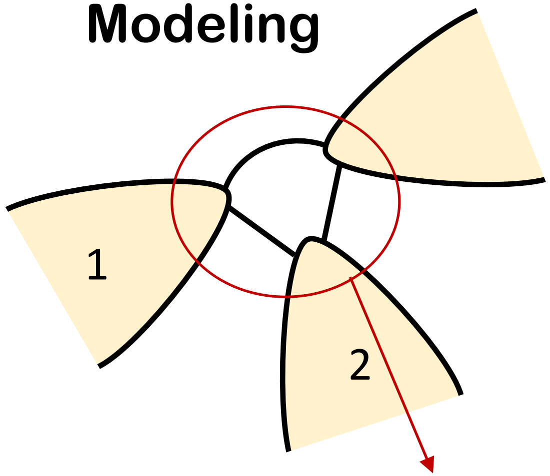






Alternative modeling: Hamiltonian approach

Modeling



$$H = H_1 + H_2 + \ldots + H_T$$

BCS-junctions, Cuevas et al. PRB 96
Difussive FS, Bergeret et al. PRB 05
Graphene/S, Burset et al. PRB 08
Topological, Zazunov et al. PRB 16
Alvarado et al. PRB 20
ABS dynamics, Seoane et al. PRL 16
.
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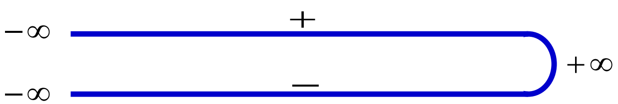
Methods



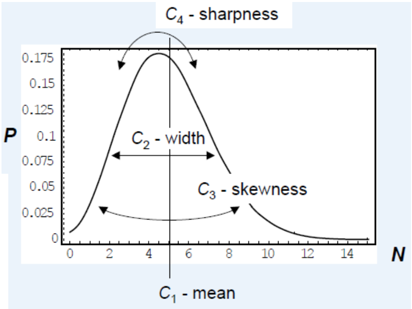
Keldysh-Nambu formalism
Non-equililbrium GFs

Aim: calculate

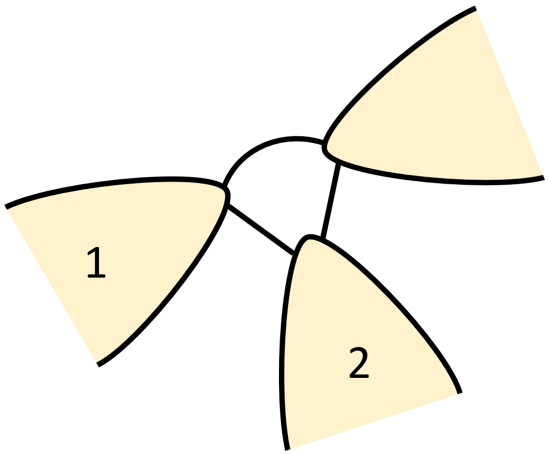
$$< I_j >$$

$$< I_i(t) I_j(t') >$$

FCS…

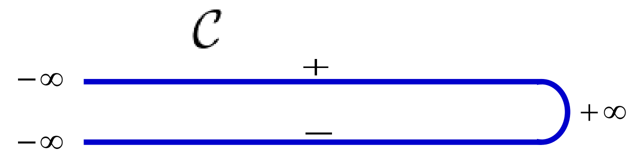


Keldysh Nambu formalism



$$\hat{\Psi}_i = \begin{pmatrix} c_{i\uparrow} \\ c^{\dagger}_{i\downarrow} \end{pmatrix}$$ Nambu spinors

Keldysh contour



Keldysh-Nambu GFs $$\hat{G}_{i,j}(t,t') = -i\langle T_{\mathcal{C}} \hat{\Psi}_i(t) \hat{\Psi}^{\dagger}_j(t') \rangle$$

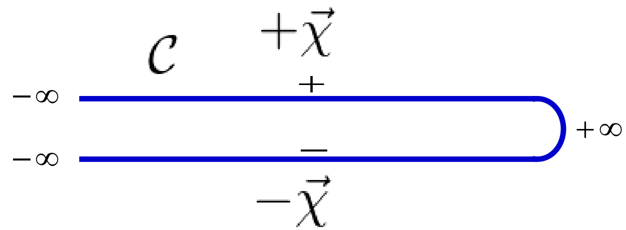
$$H_T = \sum_{ij} \hat{\Psi}^{\dagger}_i \hat{T}_{ij} \hat{\Psi}_j + \mathrm{h.}c. \qquad \hat{T}_{ij} = \begin{pmatrix} T_{ij} & 0 \\ 0 & -T^{*}_{ij} \end{pmatrix}$$

Keldysh-Nambu-Leads Dyson Eqs $$\check{\hat{G}} = \check{\hat{g}} + \check{\hat{g}} \otimes \check{\hat{\Sigma}} \otimes \check{\hat{G}}$$

Mean currents $$\langle I_{ij} \rangle(t) = \frac{e}{\hbar} \mathrm{T}r \left[\sigma_z \left(\hat{T}_{ij} \hat{G}^{+-}_{ij}(t,t) - \hat{T}^{\dagger}_{ij} \hat{G}^{+-}_{ji}(t,t) \right) \right]$$

Functional integral representation

$$H_T(\vec{\chi}) = \sum_{ij} \hat{\Psi}_i^\dagger \hat{T}_{ij}(\vec{\chi}) \hat{\Psi}_j + \mathrm{h.c.} \qquad \hat{T}_{ij}(\vec{\chi}) = T_{ij} \begin{pmatrix} e^{i\chi_{ij}} & 0 \\ 0 & -e^{-i\chi_{ij}} \end{pmatrix}$$



$$Z(\vec{\chi}) = \langle e^{-i \int_{\mathcal{C}} H_T(\vec{\chi}) dt} \rangle$$

Partition or Generating function

$$Z(\vec{\chi}) = \int \mathcal{D}\bar{\hat{\Psi}} \mathcal{D}\hat{\Psi} e^{iS_{eff}(\bar{\hat{\Psi}}, \hat{\Psi}, \vec{\chi})}$$

Boundary Green functions

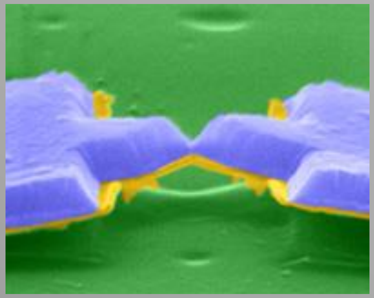
$$S_{eff}(\bar{\hat{\Psi}}, \hat{\Psi}, \vec{\chi}) = \sum_{ij} \int_{\mathcal{C}} dt \left(\bar{\hat{\Psi}}_i, \bar{\hat{\Psi}}_j \right) \begin{pmatrix} \hat{g}_i^{-1} & -\hat{T}_{ij} \\ -\hat{T}_{ij}^\dagger & \hat{g}_j^{-1} \end{pmatrix} \begin{pmatrix} \hat{\Psi}_i \\ \hat{\Psi}_j \end{pmatrix}$$

$$S(\vec{\chi}) = \log Z(\vec{\chi}) \qquad \langle I_{ij} \rangle \propto \frac{i}{2} \frac{\delta S}{\delta \chi_{ij}} \qquad \langle I_{ij} I_{kl} \rangle \propto \left(\frac{i}{2} \right)^2 \frac{\delta^2 S}{\delta \chi_{ij} \delta \chi_{kl}}$$

Cumulant Generating function

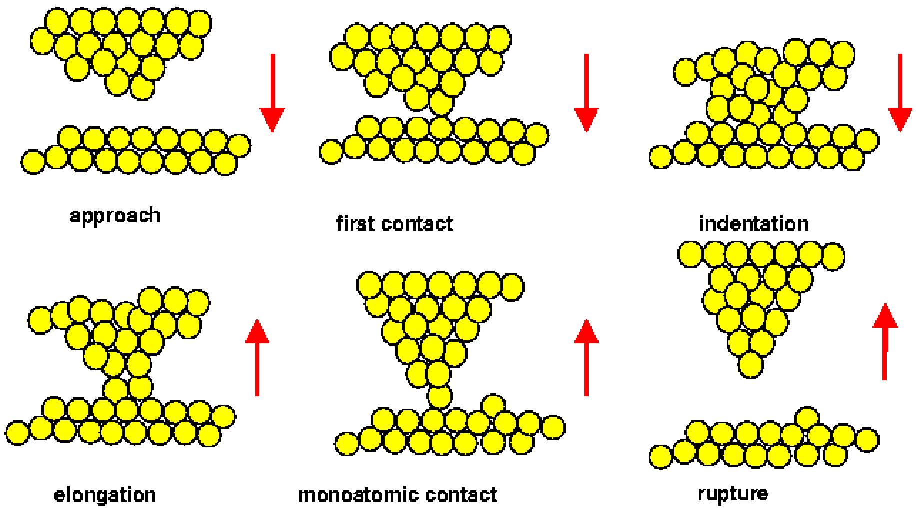
Superconducting Atomic Contacts:

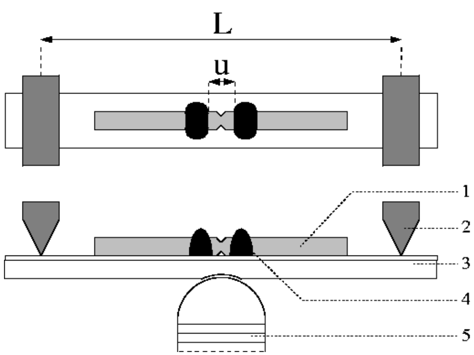
A test bed for mesoscopic superconductivity



Fabrication techniques

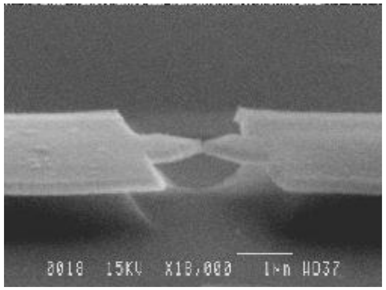
Contact formation with STM

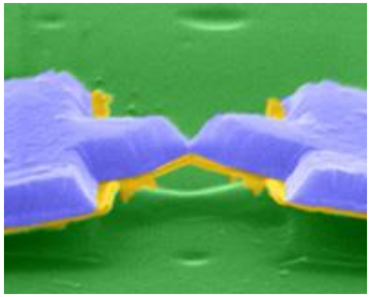




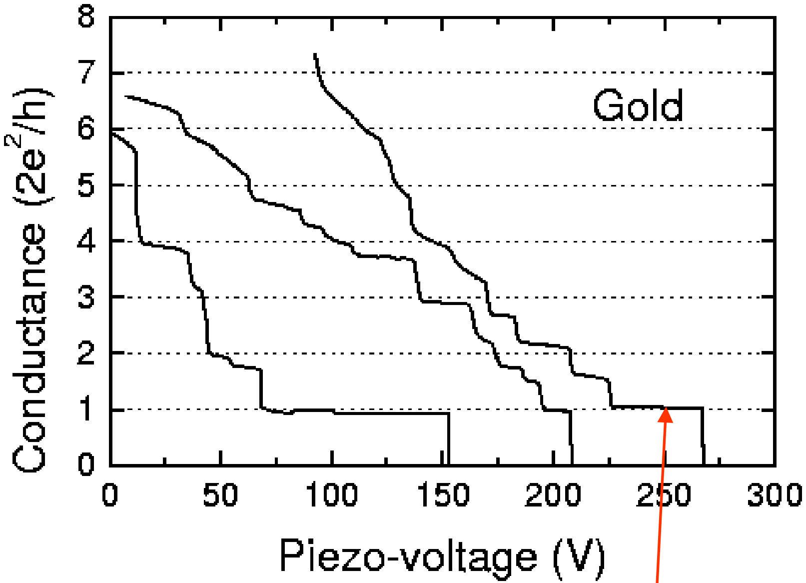
Mechanically controlable break-junctions (MCBJ)

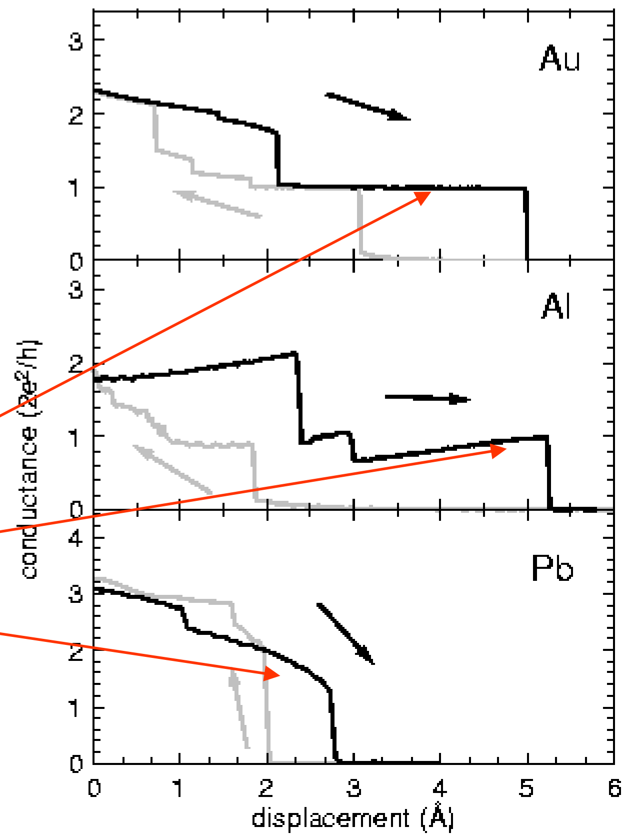
Nanofabricated break-junctions





Conductance steps





one atom limit

Results for one-atom

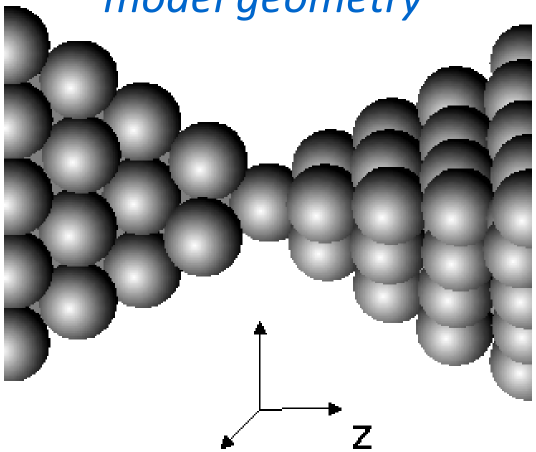
atomic configurations

Au → $5d^{10}\, 6s^1$

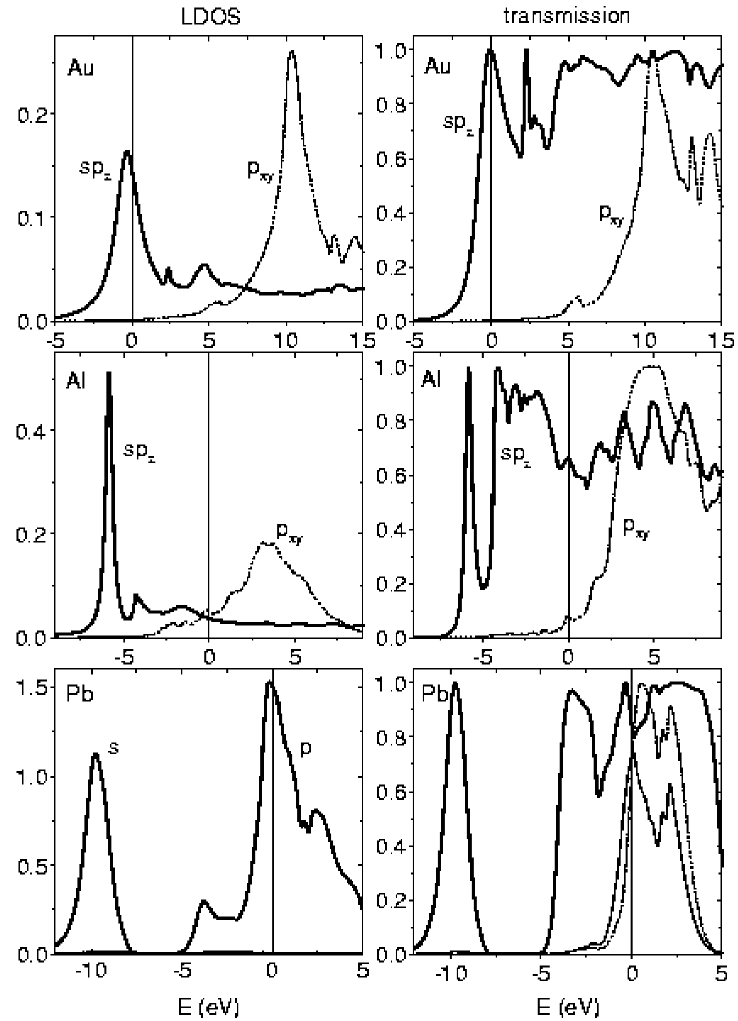
Al → $3s^2\, 3p^1$

Pb → $4s^2\, 4p^2$

model geometry

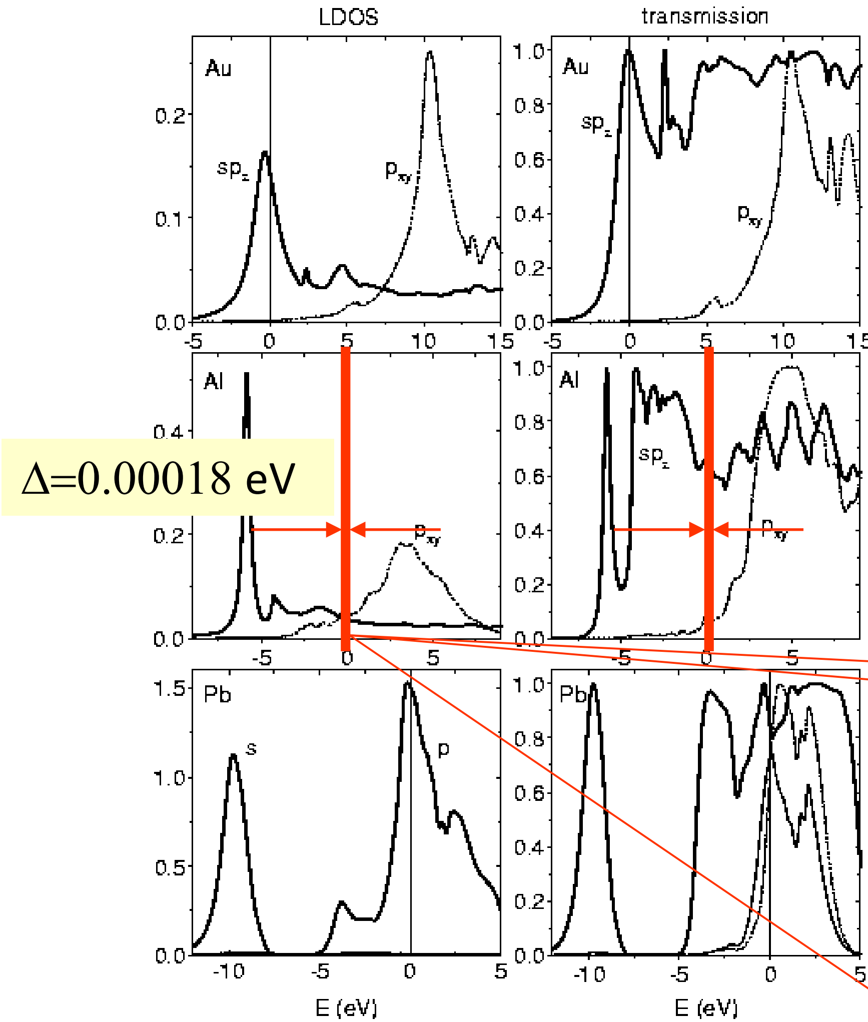


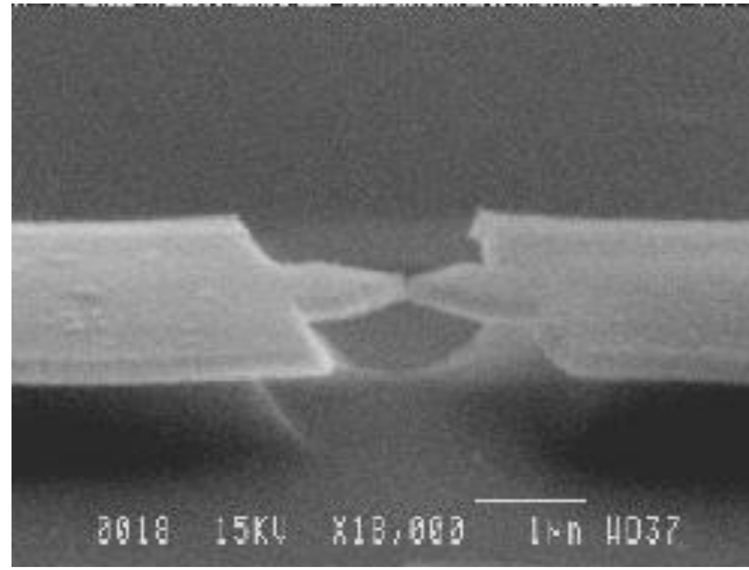
theoretical results



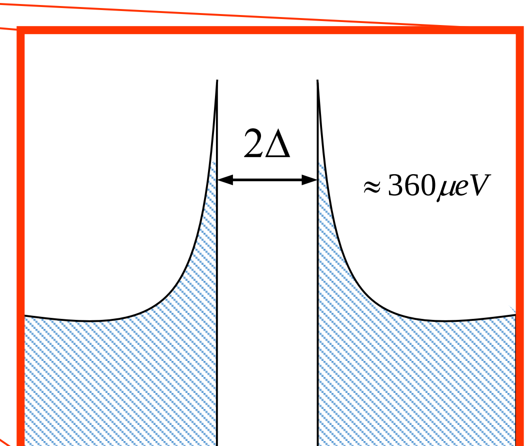
J.C. Cuevas, A. Levy Yeyati and A. Martín-Rodero, PRL 80, 1066 (1998)

Energy Scales

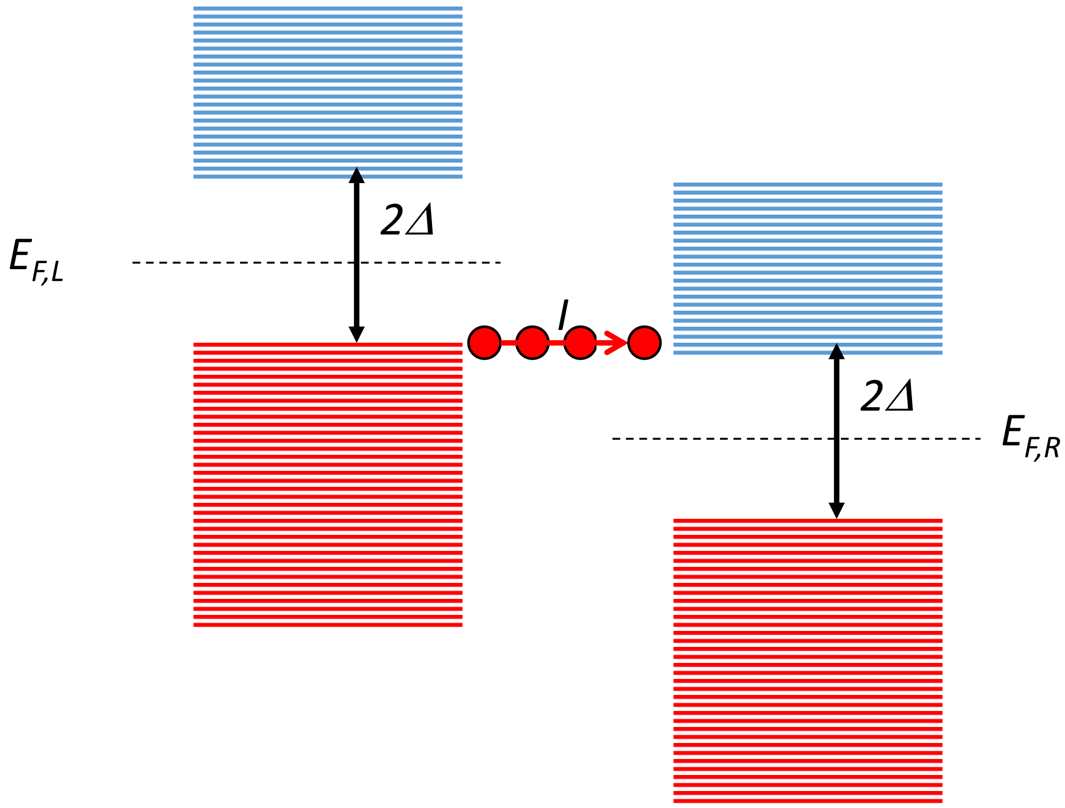


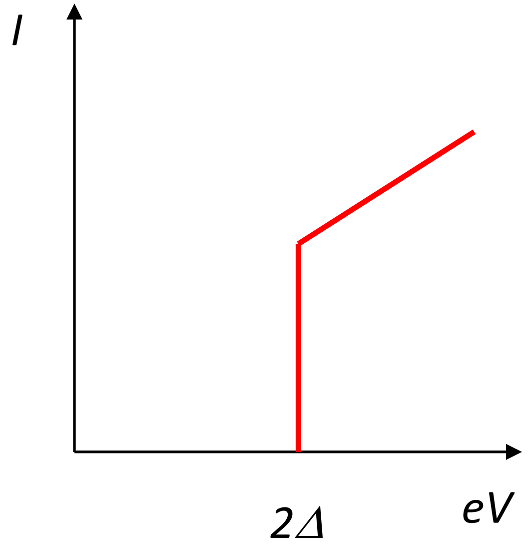


conduction channels not affected!



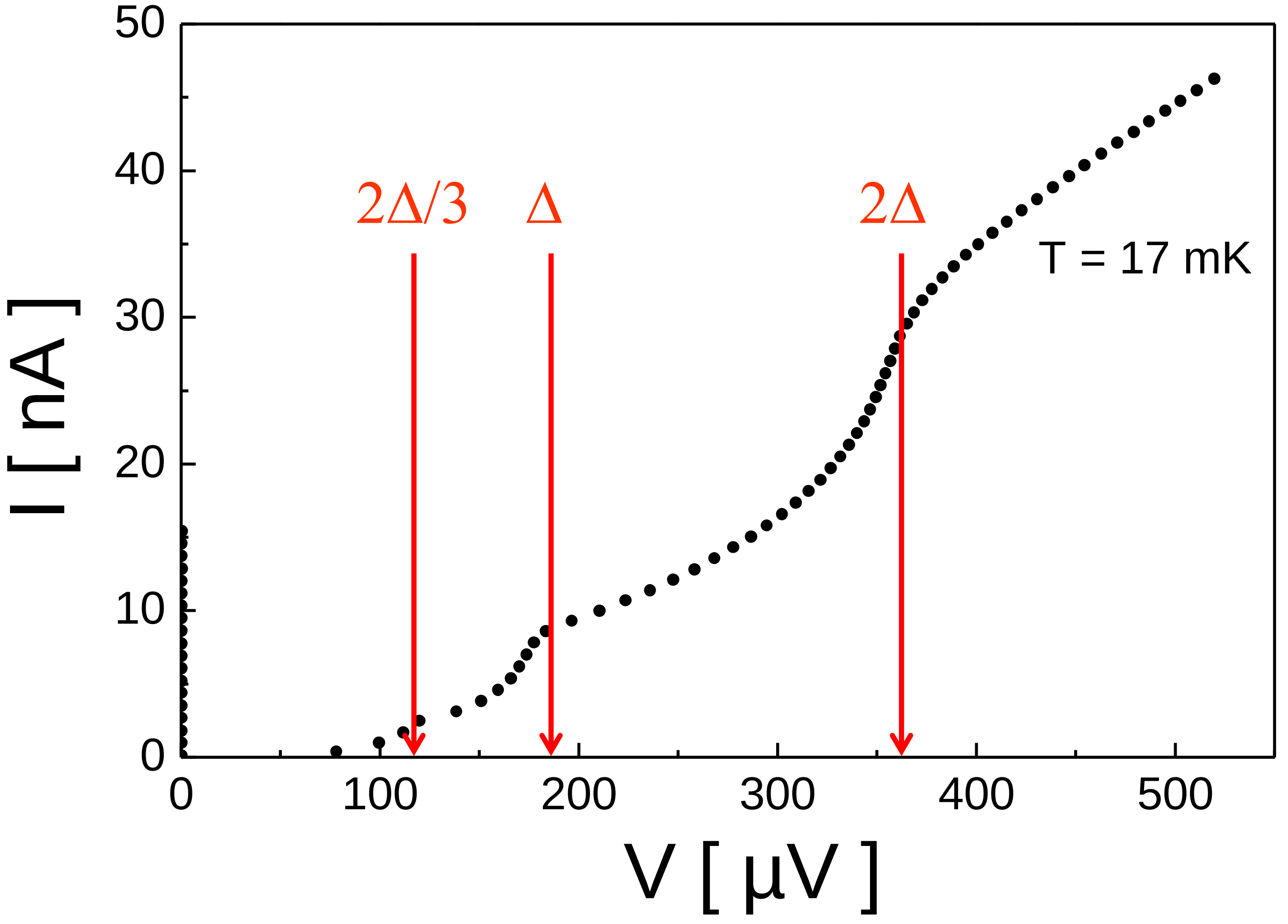
Transport between superconducting electrodes





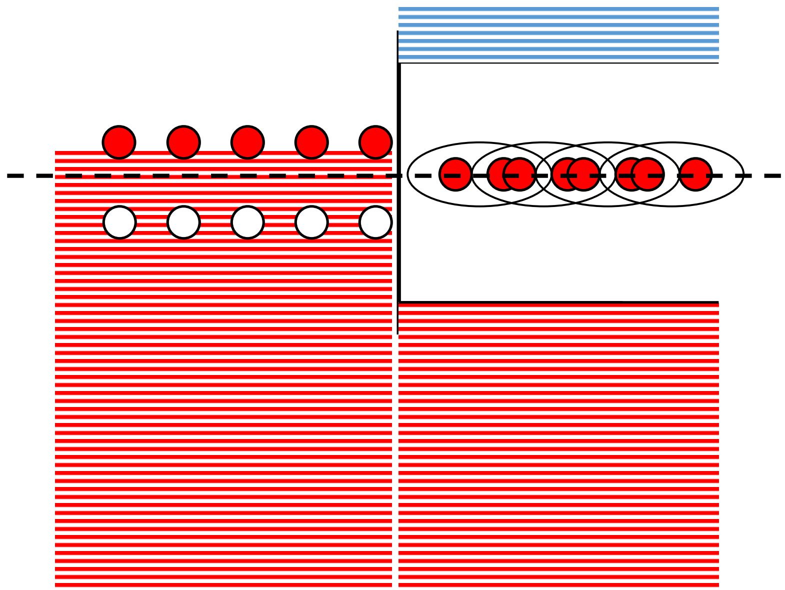
$E_{F,L} - E_{F,R} = eV > 2\Delta$

Experimental IV curves in superconducting contacts



one-atom Al contact

Andreev Reflection

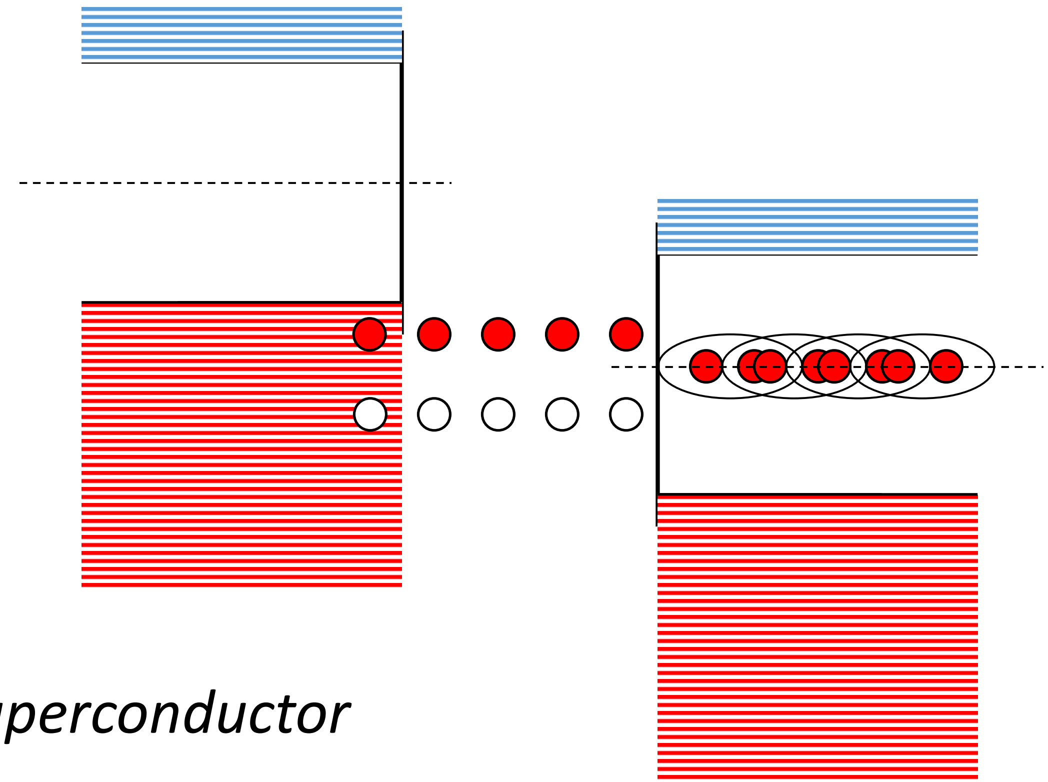


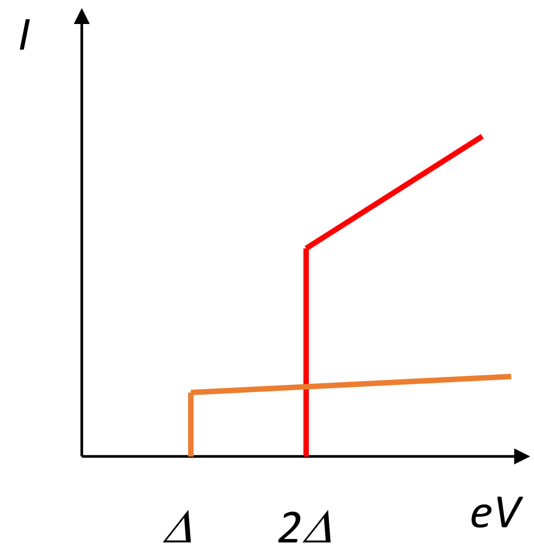
Normal metal

Superconductor

Transmitted charge $2e$ Probability $\approx \tau^2$

Andreev reflection between superconducting electrodes





Superconductor

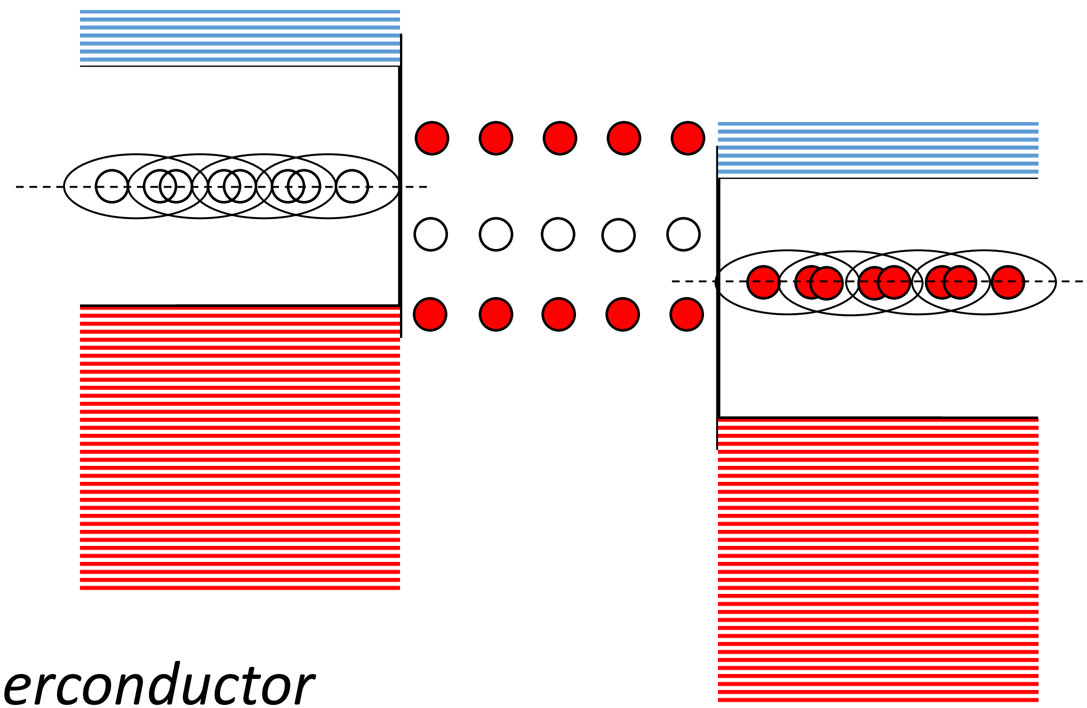
$eV > \Delta$

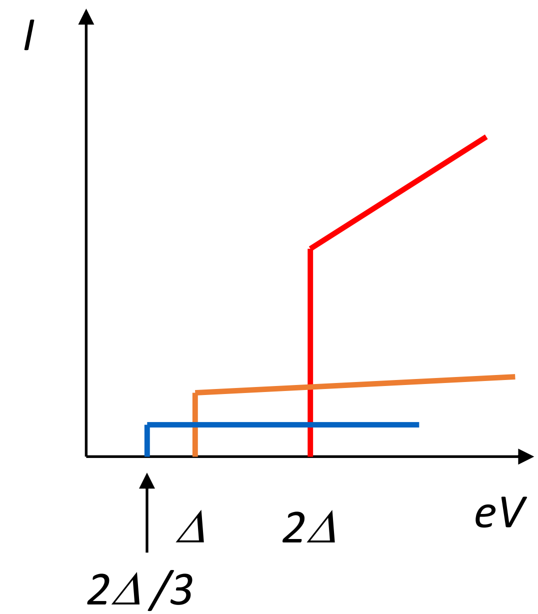
Superconductor

probability τ^2

transmitted charge $2e$

Multiple Andreev Reflection





Superconductor

Superconductor

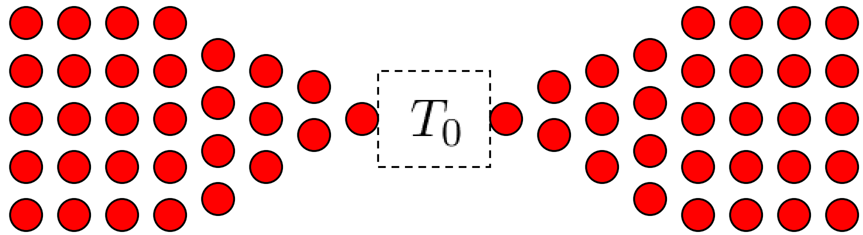
$eV > 2\Delta/3$

probability τ^3

transmitted charge $3e$

Model for single channel contact

Left lead



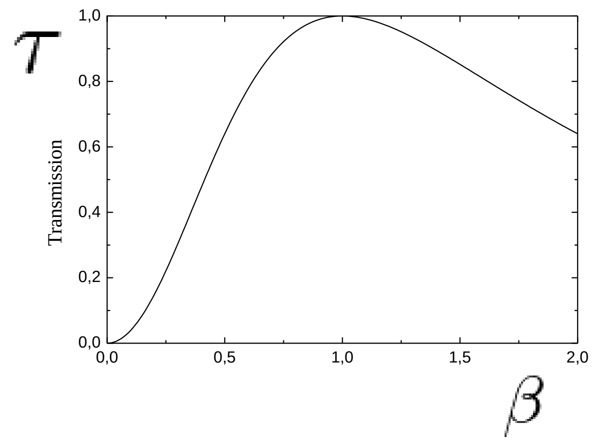
Right lead

$$H_{contact} = H_L + H_R + \underbrace{\sum_\sigma T_0 \left(c^\dagger_{L\sigma} c_{R\sigma} + \text{h.c.} \right)}_{H_T}$$

normal case $\quad H_{L,R} \rightarrow \rho(\omega) \simeq \dfrac{1}{\pi W}$

transmission coefficient →

$$\tau = \frac{4\beta}{(1+\beta)^2} \qquad \beta = \left(\frac{T_0}{W} \right)^2$$



J.C. Cuevas, A. Martín-Rodero and A. Levy Yeyati, PRB 54, 7366 (1996)

BCS leads

$$H_{BCS} = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_k \Delta c^\dagger_{k\uparrow} c_{-k\downarrow} + \mathrm{H}.c. \longrightarrow$$ *bulk leads*

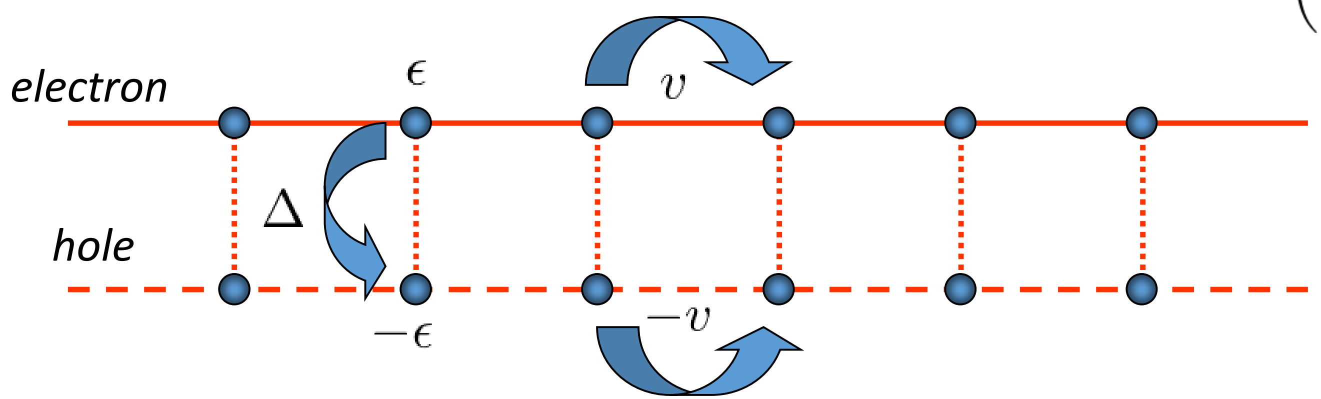
Local basis

$$H_{L,R} = \sum_{i\sigma} \epsilon_i c^\dagger_{i\sigma} c_{i\sigma} + v c^\dagger_{i\sigma} c_{i\pm 1\sigma} + \sum_i \Delta_i c^\dagger_{i\uparrow} c_{i\downarrow} + \mathrm{h.c.}$$

Nambu rep

$$H_{L,R} = \sum_i \Psi^\dagger_i \hat{\epsilon}_i \Psi_i + \Psi^\dagger_i \hat{v} \Psi_{i\pm 1} \qquad \Psi_i = \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix}$$

$$\hat{\epsilon}_i = \begin{pmatrix} \epsilon_i & \Delta_i \\ \Delta^*_i & -\epsilon_i \end{pmatrix} \qquad \hat{v} = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$$



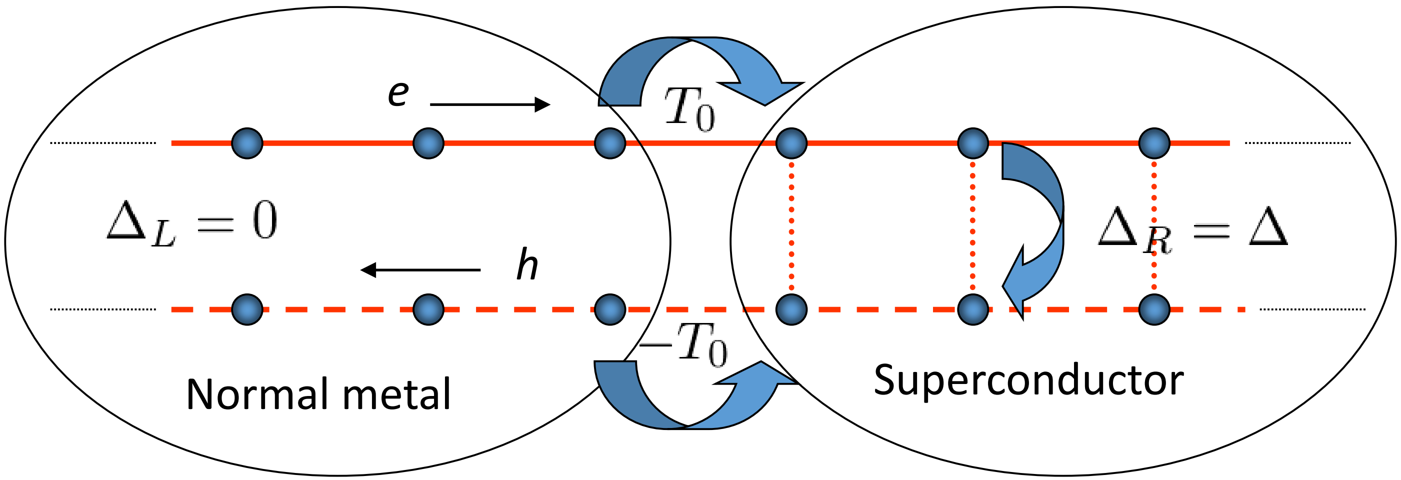
Boundary GFs

$$\omega, \Delta \ll v$$

$$\hat{g}^{r,a}(\omega) = [\omega \pm i0 - \hat{\epsilon} - \hat{v}\hat{g}^{r,a}(\omega)\hat{v}]^{-1}$$

$$\hat{g}^{r,a}(\omega) \simeq \frac{1}{v} \left(\frac{-(\omega \pm i0)\sigma_0 + \Delta\sigma_x}{\sqrt{\Delta^2 - (\omega \pm i0)^2}} \right)$$

Coupling two leads: NS interface

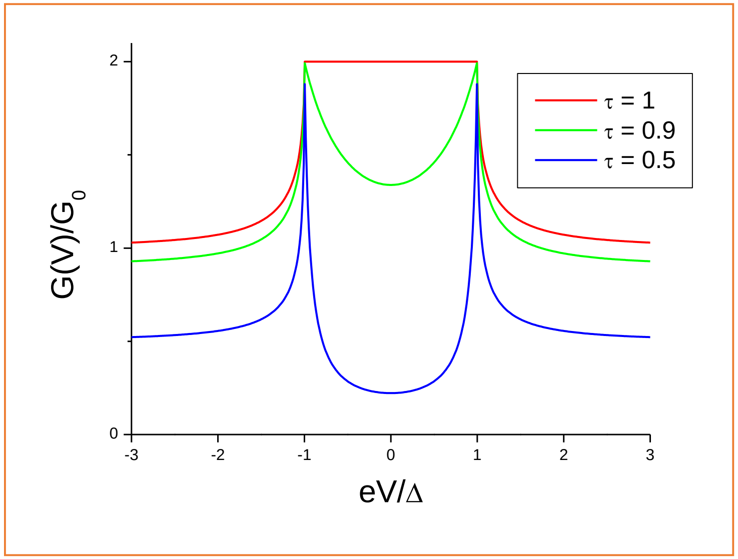


Andreev reflection

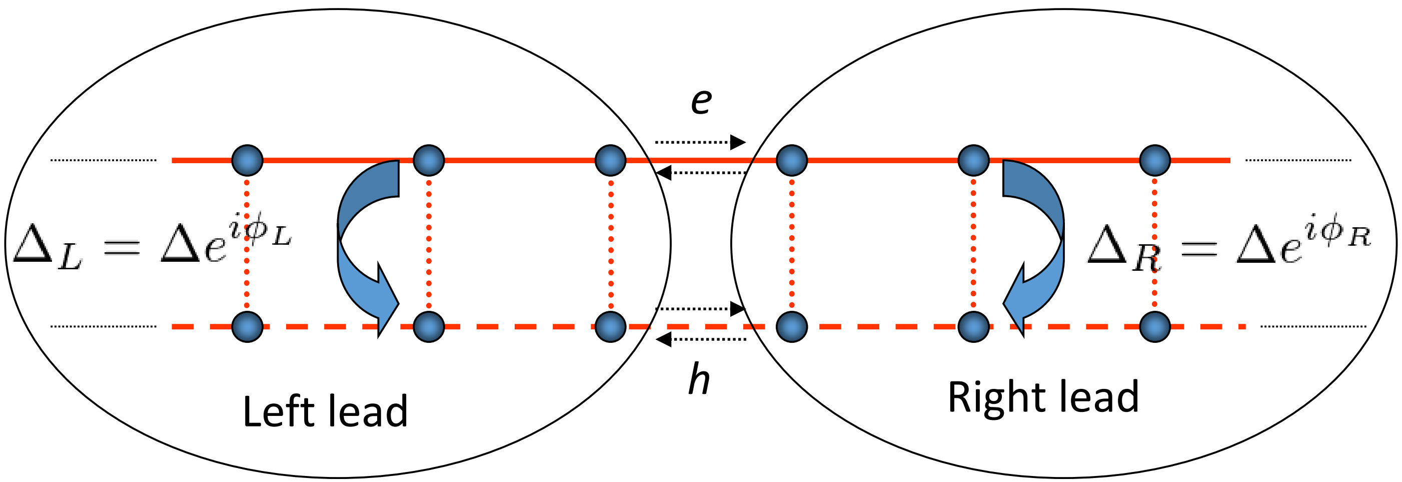
$$R_A(E) = \frac{\tau^2 \Delta^2}{\tau^2 E^2 + (\Delta^2 - E^2)(2-\tau)^2} \quad |E| \leq \Delta$$

$$G_{NS}(V) = \frac{4e^2}{h} R_A(eV) \quad e|V| \leq \Delta$$

Blonder, Tinkham & Klapwijk, PRB 25, 4515 (1982)

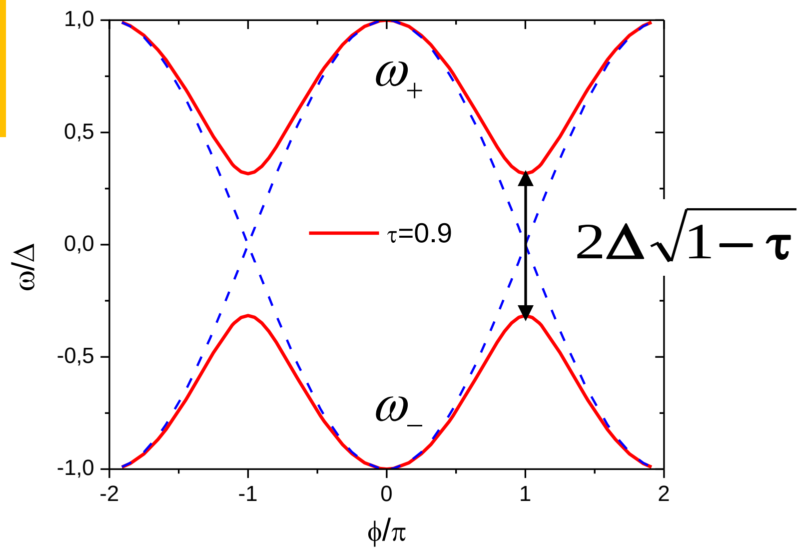


Coupling two superconductors: relevance of phase difference



$$\phi = \phi_L - \phi_R$$

$$\omega_\pm = \pm\Delta\sqrt{1 - \tau \sin^2\left(\frac{\phi}{2}\right)}$$



supercurrent (T=0)

$$I = \frac{e}{\hbar}\frac{\partial \omega_-}{\partial \phi} = \frac{e\Delta\tau}{2\hbar}\frac{\sin(\phi)}{\sqrt{1 - \tau \sin^2\left(\frac{\phi}{2}\right)}}$$

$I \propto \sin(\phi)$

$\tau \to 0$

$I \propto \mathrm{sgn}(\pi - \phi)\sin\left(\frac{\phi}{2}\right) \quad (\tau \to 1)$

Voltage biased SC contact: intrinsic time dependence

$$H_{L,R} \to H_{L,R} - \mu_{L,R} N_{L,R} \qquad eV = \mu_L - \mu_R$$

$$\phi_{L,R} \to \phi_{L,R} + \int dt \frac{2e\mu_{L,R}}{\hbar} \qquad \Rightarrow \frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar} = \omega_0$$

Josephson frequency

Gauge transformation: eliminate time dependence from leads

$$H_T \to H_T = \sum_\sigma T_0 \left(c^\dagger_{L\sigma} c_{R\sigma} e^{i\phi(t)/2} + c^\dagger_{R\sigma} c_{L\sigma} e^{-i\phi(t)/2} \right)$$

Nambu form $\quad H_T = \Psi^\dagger_L \hat{T}_{LR}(t) \Psi_R + \Psi^\dagger_R \hat{T}_{RL} \Psi_L$

$$\hat{T}_{LR} = T_0 \begin{pmatrix} e^{i\phi(t)/2} & 0 \\ 0 & -e^{-i\phi(t)/2} \end{pmatrix} = T^*_{RL}$$

Current operator

$$I(t) = \frac{ie}{\hbar} \left[\Psi^\dagger_L \hat{T}_{LR}(t) \Psi_R - \text{h.c.} \right]$$

$$\langle I \rangle (t) = \frac{e}{\hbar} \mathrm{Tr} \left[\sigma_z \left(\hat{T}_{LR} \hat{G}^{+-}_{LR}(t,t) - \hat{T}^*_{LR} \hat{G}^{+-}_{RL}(t,t) \right) \right]$$

Coupled integral equations for full GFs

$$\hat{G}^{r,a} = \left(\hat{1} + \hat{G}^{r,a} \otimes \hat{\Sigma}^{r,a}\right) \otimes \hat{g}^{r,a}$$

$$\hat{\Sigma}^{r,a}_{LR} = \left(\hat{\Sigma}^{r,a}_{RL}\right)^* = \hat{T}_{LR}(t)$$

$$\hat{G}^{+-} = \left(\hat{1} + \hat{G}^{r} \otimes \hat{\Sigma}^{r}\right) \otimes \hat{g}^{+-} \otimes \left(\hat{1} + \hat{\Sigma}^{a} \otimes \hat{G}^{a}\right)$$

Double Fourier transformation

$$\hat{G}(t,t') = \frac{1}{2\pi} \int \int d\omega d\omega' e^{-i\omega t + i\omega' t'} \hat{G}(\omega,\omega')$$

$$\hat{G}(\omega,\omega') \rightarrow \sum_n \hat{G}_{n0}(\omega) \delta\left(\omega - \omega' + n\frac{\omega_0}{2}\right)$$

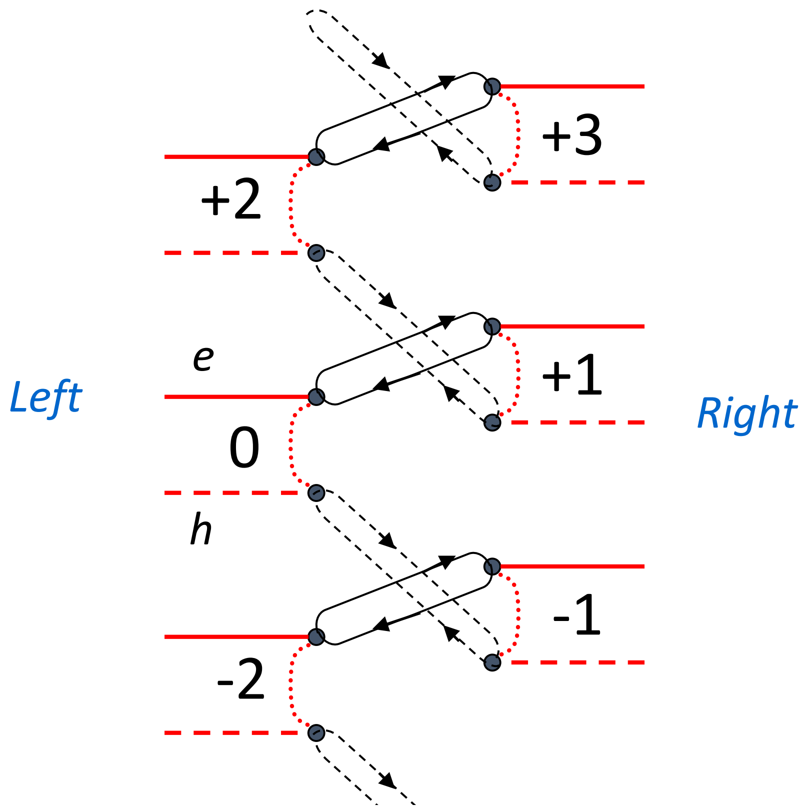
$$\langle I \rangle(t) = \frac{e}{\hbar} \mathrm{Tr}\left[\sigma_z \left(\hat{T}_{LR} G^{+-}_{LR}(t,t) - \hat{T}^*_{LR} G^{+-}_{RL}(t,t)\right)\right]$$

$$\Longrightarrow I(t,V) = \sum_n I_n(V) e^{in\omega_0 t}$$

dc + ac components

Pictorial representation

$$\hat{\mathbf{T}}^{+}_{\mathbf{LR}} = \begin{pmatrix} \mathbf{T}_0 & 0 \\ 0 & 0 \end{pmatrix} = \hat{\mathbf{T}}^{-}_{\mathbf{RL}} \qquad \hat{\mathbf{T}}^{-}_{\mathbf{LR}} = \begin{pmatrix} 0 & 0 \\ 0 & -\mathbf{T}_0 \end{pmatrix} = \hat{\mathbf{T}}^{+}_{\mathbf{RL}}$$



$$\hat{\mathbf{G}}_{00}(\omega) = \left[\hat{\mathbf{g}}_0^{-1} - \hat{\mathbf{T}}^{+}_{\mathbf{LR}} \hat{\mathcal{G}}_1 \hat{\mathbf{T}}^{-}_{\mathbf{RL}} - \hat{\mathbf{T}}^{-}_{\mathbf{LR}} \hat{\mathcal{G}}_{-1} \hat{\mathbf{T}}^{+}_{\mathbf{RL}} \right]^{-1}$$

$$\hat{\mathcal{G}}_1(\omega) = \left[\hat{\mathbf{g}}_1^{-1} - \hat{\mathbf{T}}^{+}_{\mathbf{RL}} \hat{\mathcal{G}}_2 \hat{\mathbf{T}}^{-}_{\mathbf{LR}} \right]^{-1}$$

$$\hat{\mathcal{G}}_{-1}(\omega) = \left[\hat{\mathbf{g}}_{-1}^{-1} - \hat{\mathbf{T}}^{-}_{\mathbf{RL}} \hat{\mathcal{G}}_{-2} \hat{\mathbf{T}}^{+}_{\mathbf{LR}} \right]^{-1}$$

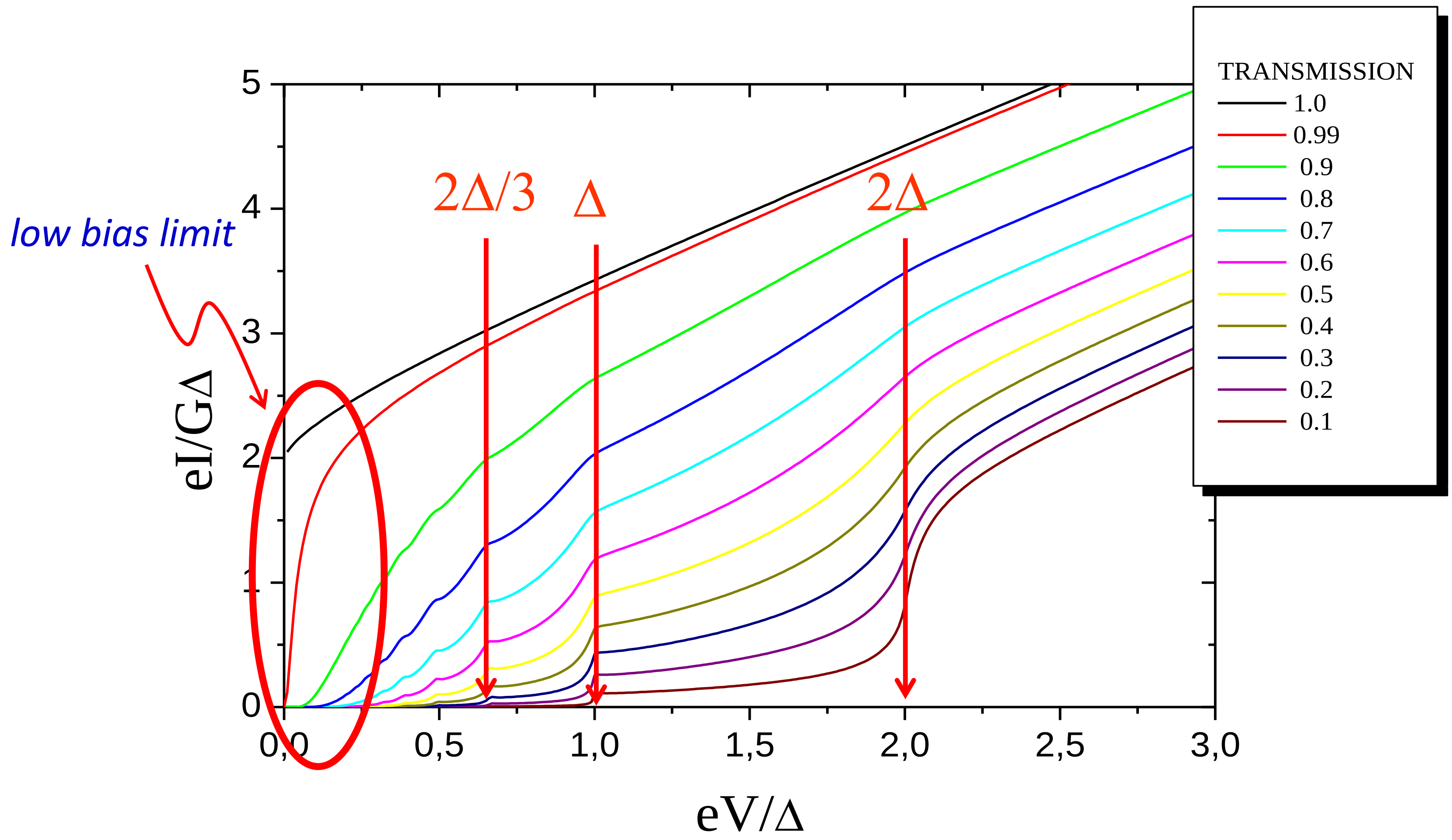
$$\vdots$$

$$\hat{\mathcal{G}}_{\mathbf{n}}(\omega) = \left[\hat{\mathbf{g}}_1^{-1} - \hat{\mathbf{T}}^{+}_{\mathbf{LR}} \hat{\mathcal{G}}_{\mathbf{n}+1} \hat{\mathbf{T}}^{-}_{\mathbf{RL}} \right]^{-1}$$

$$\hat{\mathcal{G}}_{-\mathbf{n}}(\omega) = \left[\hat{\mathbf{g}}_1^{-1} - \hat{\mathbf{T}}^{-}_{\mathbf{LR}} \hat{\mathcal{G}}_{-\mathbf{n}-1} \hat{\mathbf{T}}^{+}_{\mathbf{RL}} \right]^{-1}$$

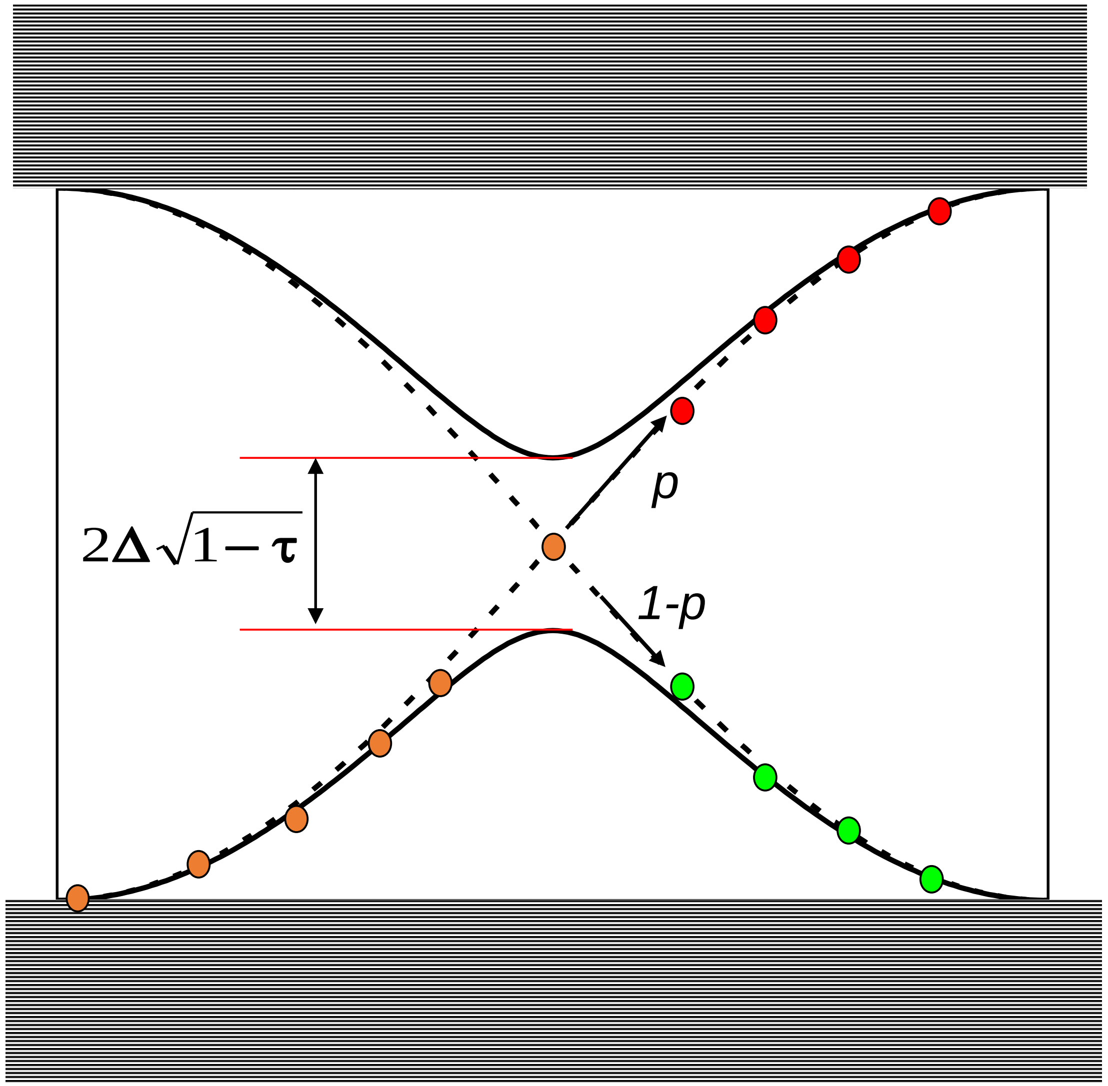
truncation $\boxed{n >> 2\Delta / V}$

Theoretical IV curves for superconducting contacts



J.C. Cuevas, A. Martín-Rodero and A. Levy Yeyati, PRB 54, 7366 (1996)
same results with different approach: Averin & Bardas (95), Shumeiko et al. (97)

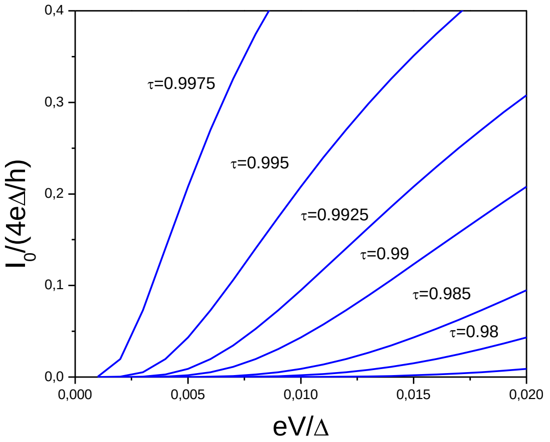
Landau-Zener transitions between AS's



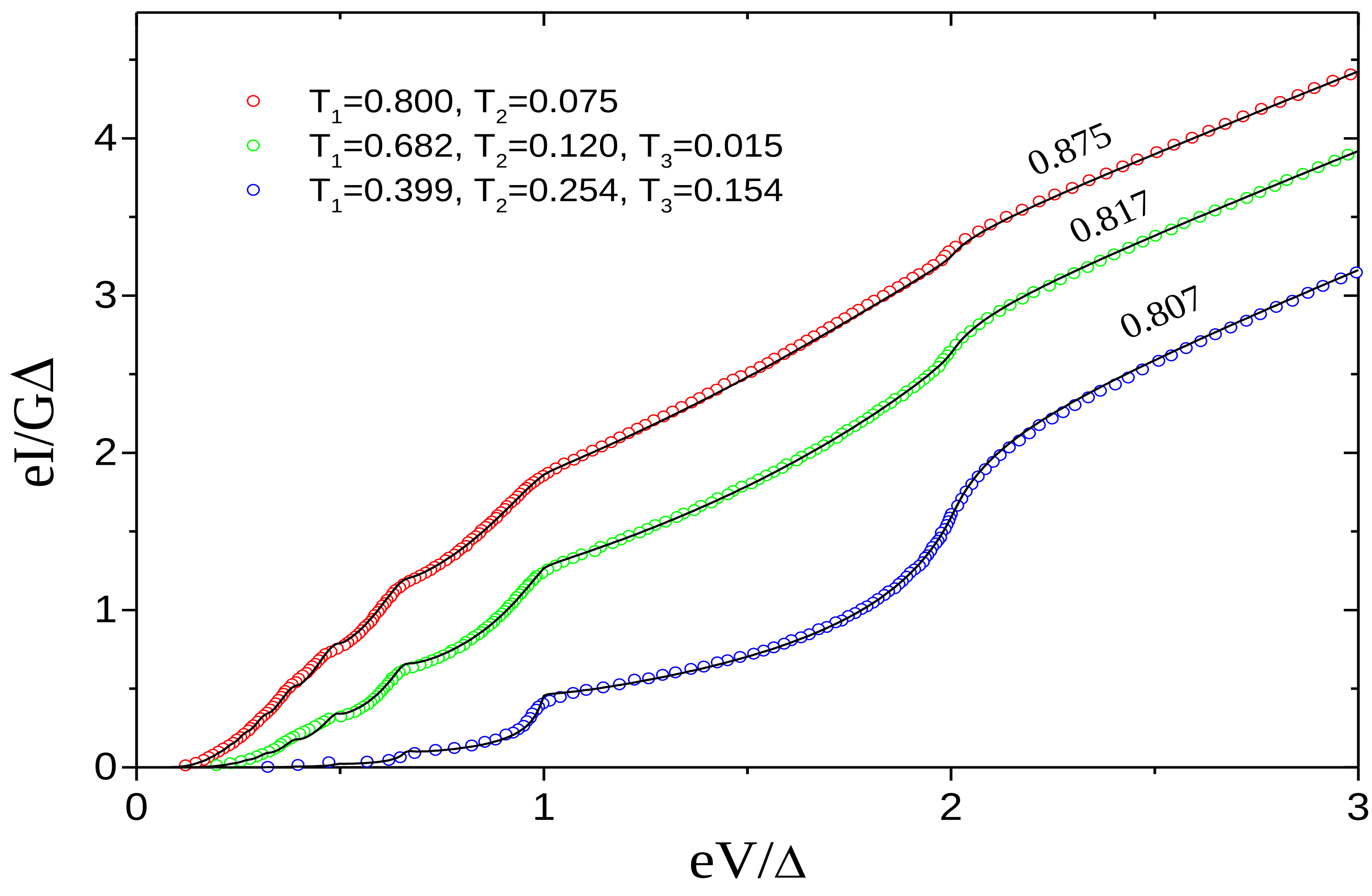
$$p = \exp\left(-\frac{\pi \Delta R}{eV}\right)$$

$$R = 1 - \tau$$

$$I_0(V) \simeq \frac{4e\Delta}{h} p$$

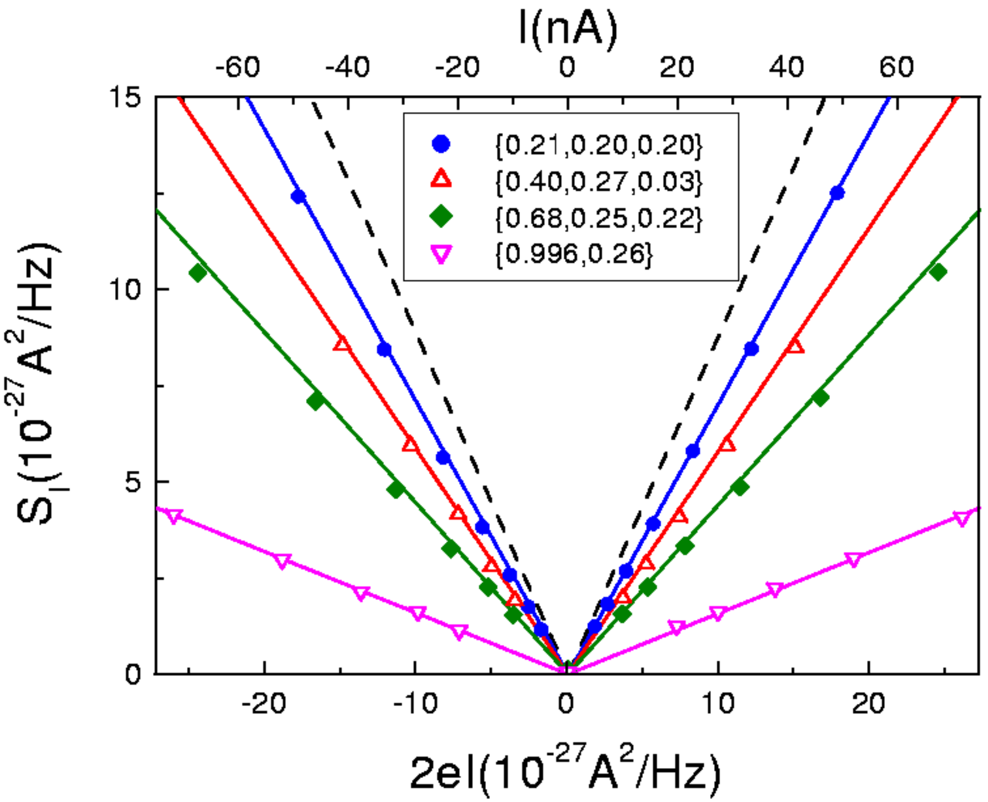


Fitting IV curves for Al contacts



E. Scheer et al, PRL 1997

Consistency with noise measurements



R. Cron et al, PRL 2001

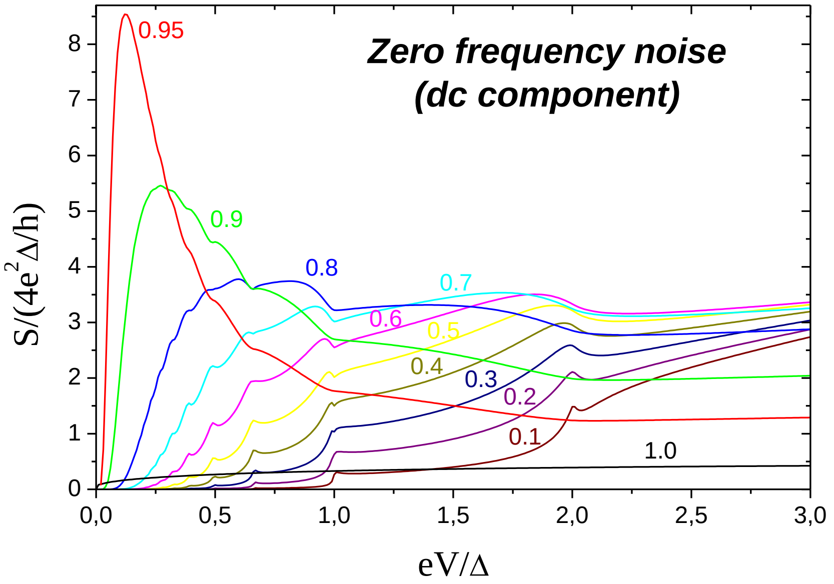
$$S(V) = \int \langle I(t)I(0)\rangle - \langle I\rangle^2 dt = 2eV\,\frac{2e^2}{h}\sum_n \tau_n(1-\tau_n)$$

Shot noise in superconducting contacts

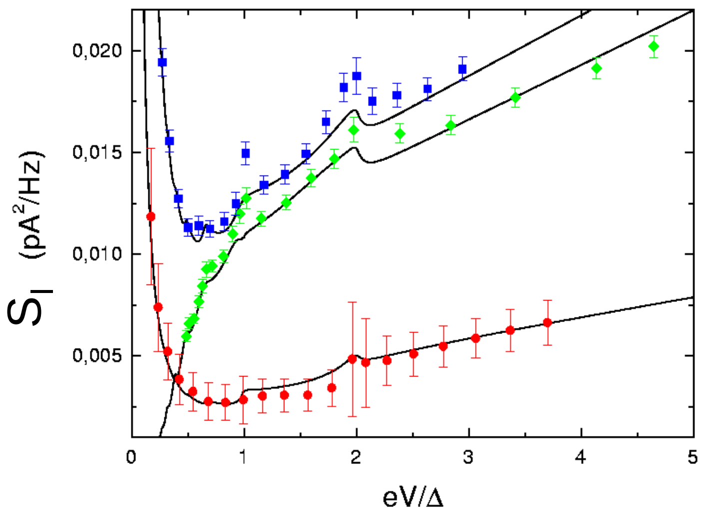
J.C.C, A.M.R and A.L.Y.
Y. Naveh and D. Averin } *PRL 82 (1999)*

$$S(0) = \int dt < \delta I(t)\delta I(0) + \delta I(0)\delta I(t) >$$

$$\delta I(t) = I(t) - < I(t) >$$



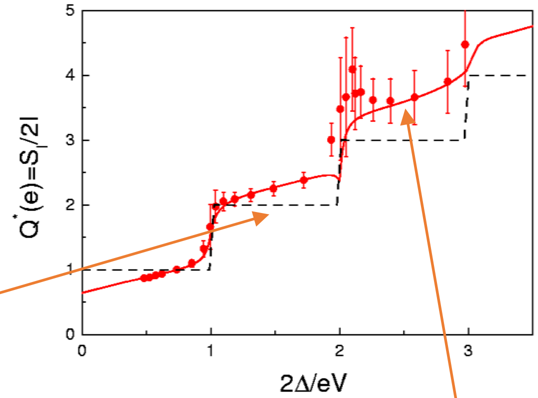
Comparison to experiments
(no adjustable parameters!)



R. Cron et al, PRL 2001

Effective charge $$Q^* = S(0)/2eI$$

tunnel limit $$Q^* = \mathrm{Int}\left[1 + 2\Delta/eV\right]$$



experimental values for $$\tau_n = \{0.40, 0.27, 0.03\}$$

Physical Review Focus

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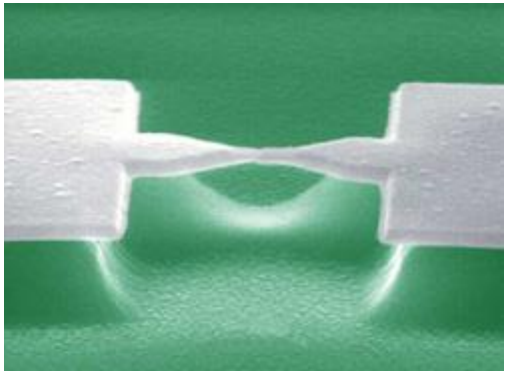
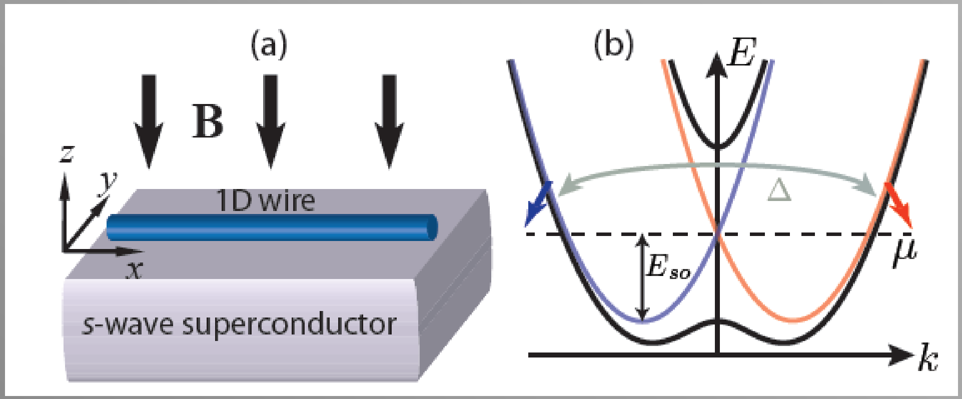
30 April 2001

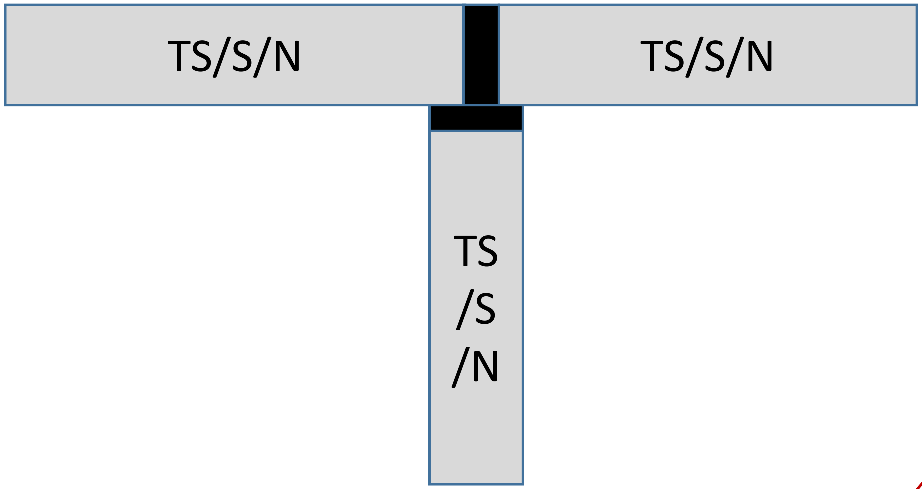
Electric Current in Big Chunks



Andreev transport and topological superconductivity



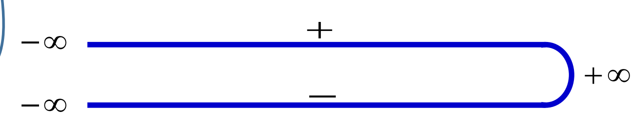
Hamiltonian approach



Appropriate bGF for TS?

$$S_{eff}\left[\hat{\bar{\Psi}}, \hat{\Psi}\right] = \sum_{ij} \int_{\mathcal{C}} dt \, \left(\bar{\Psi}_i, \; \bar{\Psi}_j\right) \begin{pmatrix} \hat{g}_i^{-1} & -\hat{T}_{ij} \\ -\hat{T}_{ji} & \hat{g}_j^{-1} \end{pmatrix} \begin{pmatrix} \Psi_i \\ \Psi_j \end{pmatrix}$$

Keldysh contour



TS case: Boundary GF for the Kitaev model

L/R chains in real space

$$H_{L/R} = \sum_{j \in L/R} t c_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + \text{h.c.}$$

infinte chain (k space, Nambu)

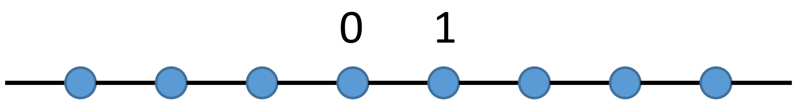
$$H_0 = \sum_k \Psi_k^\dagger \underbrace{\begin{pmatrix} t\cos k & -i\Delta \sin k \\ i\Delta \sin k & -t\cos k \end{pmatrix}}_{\mathcal{H}_k} \Psi_k \qquad \Psi_k^T = \begin{pmatrix} c_k & c_{-k}^\dagger \end{pmatrix}$$

infinite chain GF in real space

$$\hat{G}^0_{ij} = \sum_k \left[\omega - \mathcal{H}_k\right]^{-1} e^{ik|i-j|}$$

$$\hat{G}^0_{00} = \frac{-\omega}{\sqrt{(\omega^2 - \Delta^2)(\omega^2 - t^2)}}\sigma_0$$

$$\hat{G}^0_{01} = \frac{t(z_1^2 + 1) + \Delta(z_1^2 - 1)\sigma_x}{\sqrt{(\omega^2 - \Delta^2)(\omega^2 - t^2)}}\sigma_z$$

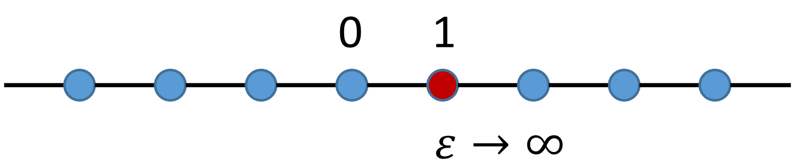


$$z_1^2 = \frac{2\omega^2 - (t^2 + \Delta^2)}{t^2 - \Delta^2} - \text{sign}\left(2\omega^2 - (t^2 + \Delta^2)\right)\sqrt{\left(\frac{2\omega^2 - (t^2 + \Delta^2)}{t^2 - \Delta^2}\right)^2 - 1}$$

Dyson equation for chain breaking

$$\hat{g}_L = \hat{G}^0_{00} - \hat{G}^0_{01}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{10}$$

$$\hat{g}_R = \hat{G}^0_{00} - \hat{G}^0_{10}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{01}$$



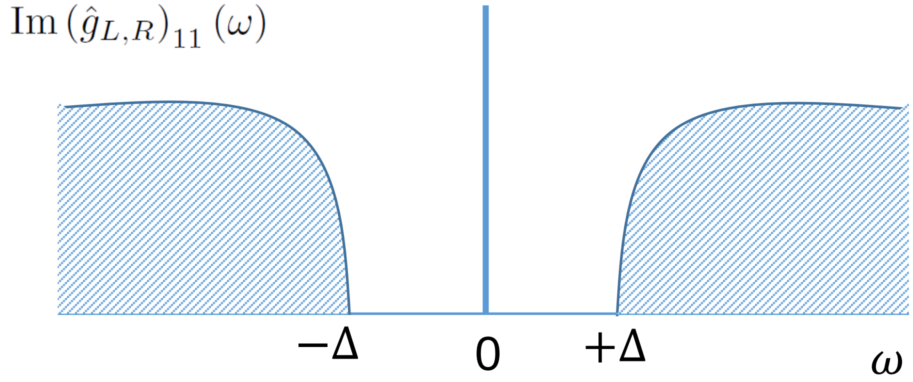
Boundary GF for the Kitaev model

Zazunov, Egger & ALY, PRB (2016)

Boundary GFs in $t >> \Delta$ *limit*

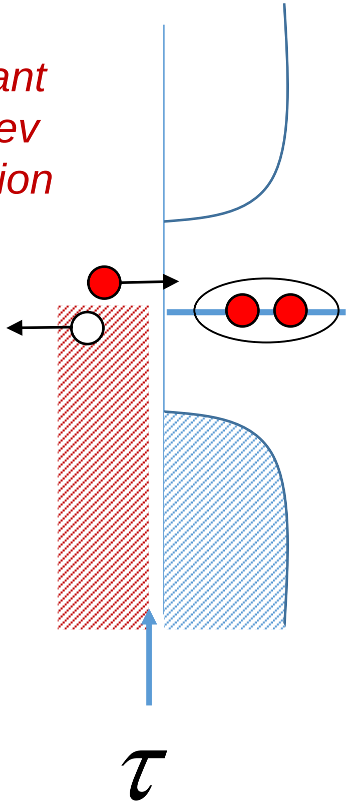
$$\hat{g}_L = \frac{2}{|t|\omega} \begin{pmatrix} \sqrt{\Delta^2 - \omega^2} & \Delta \\ \Delta & \sqrt{\Delta^2 - \omega^2} \end{pmatrix}$$

$$\hat{g}_R = \frac{2}{|t|\omega} \begin{pmatrix} \sqrt{\Delta^2 - \omega^2} & -\Delta \\ -\Delta & \sqrt{\Delta^2 - \omega^2} \end{pmatrix}$$



N-TS case: conductance and noise

Zazunov, Egger & ALY, PRB (2016)

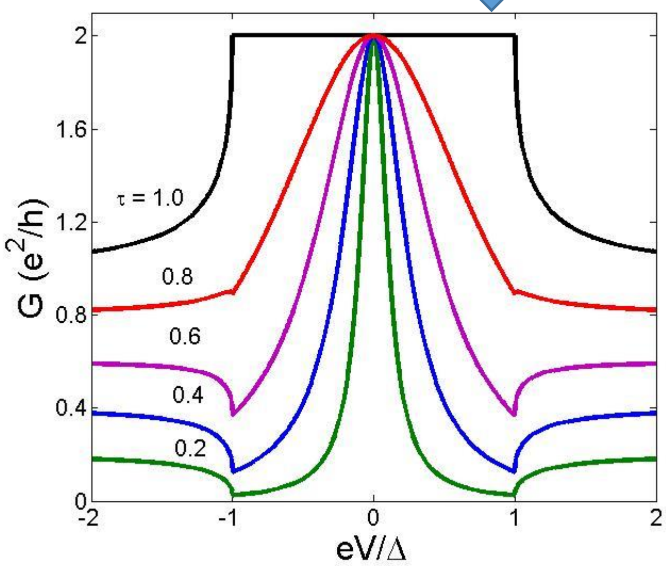


$$G(V,T=0)=\frac{2e^2}{h}J(eV)$$

zero-temperature conductance

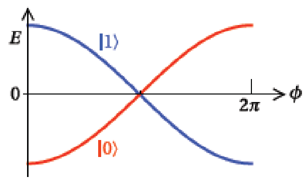
$$J(\omega)=\begin{cases}1/(1+\omega^2/\Gamma^2), & |\omega|<\Delta,\\ \tau\frac{\tau+(2-\tau)\sqrt{1-(\Delta/\omega)^2}}{\left[2-\tau+\tau\sqrt{1-(\Delta/\omega)^2}\right]^2}, & |\omega|\geq\Delta,\end{cases}$$

$$\Gamma=\frac{\tau\Delta}{2\sqrt{1-\tau}}$$



Equilibrium TS-TS case: frequency dependent noise

Zazunov, Egger & ALY, PRB (2016)



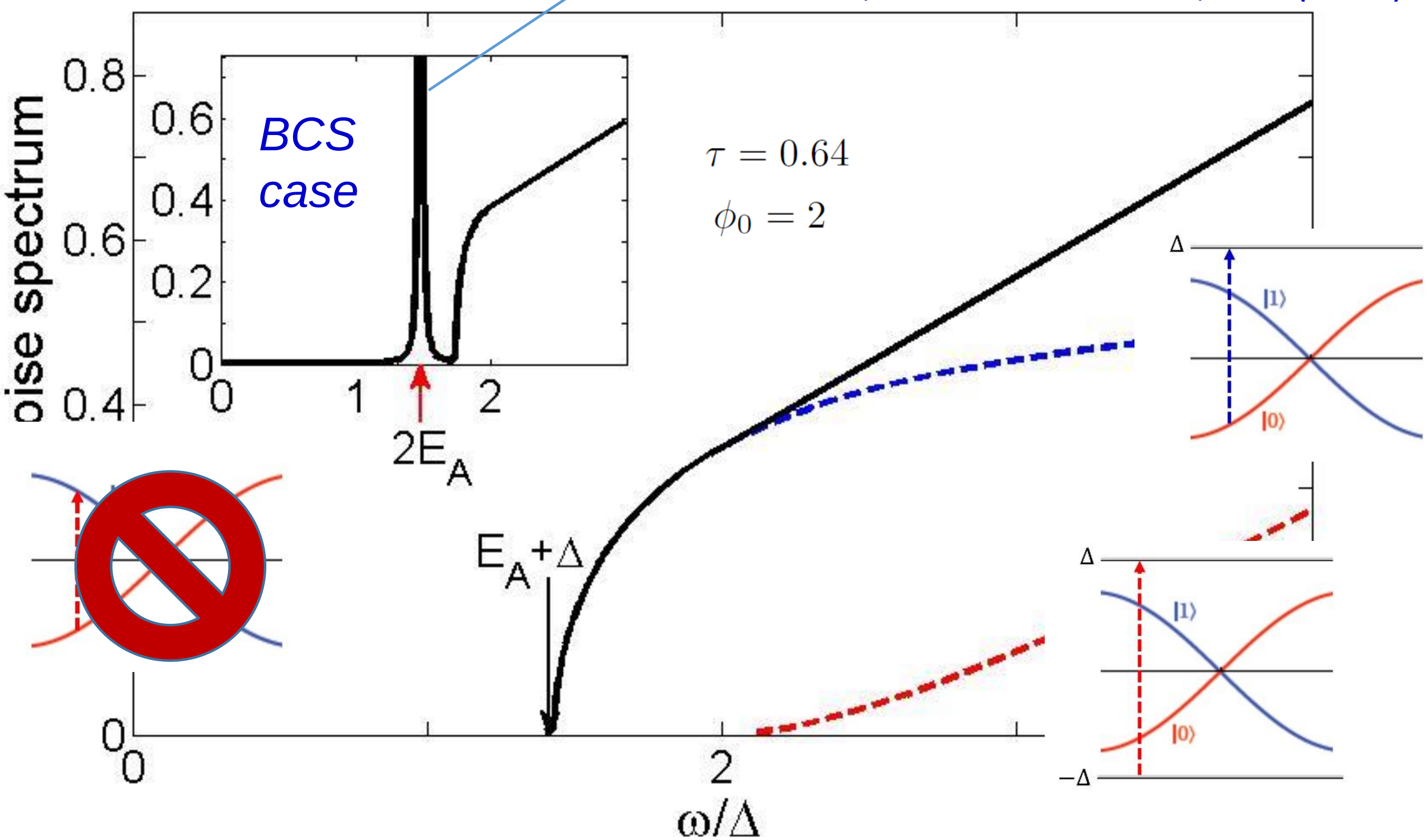
$$E_A(\phi_0) = \sqrt{\tau}\Delta\cos(\phi_0/2)$$

Andreev bound states (ABS): 4π periodicity

$$I(\phi_0) = \pm\frac{e\sqrt{\tau}\Delta}{2\hbar}\sin(\phi_0/2)$$

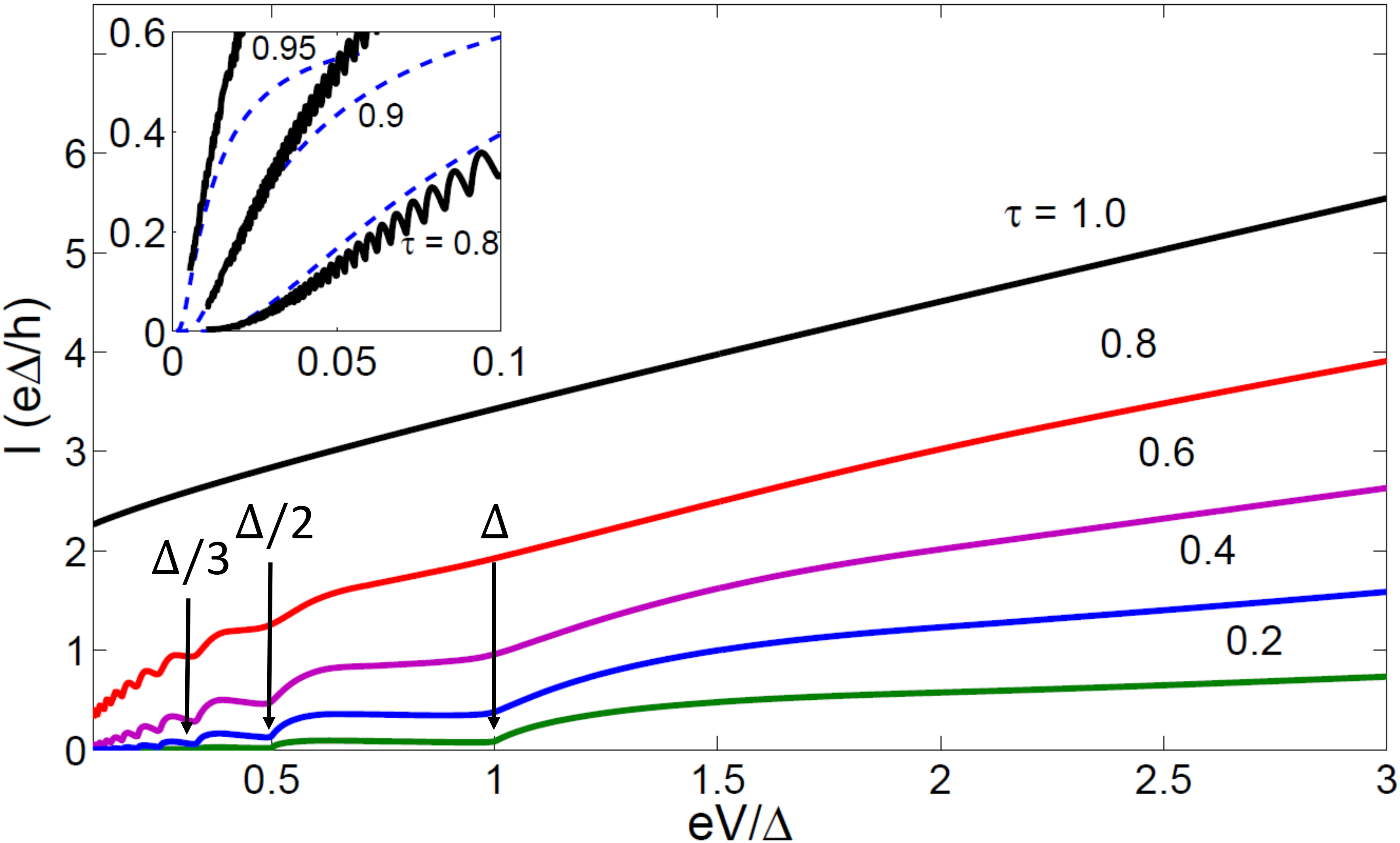
zero-temperature Josephson current

Martín-Rodero, ALY & García-Vidal, PRB (1996)



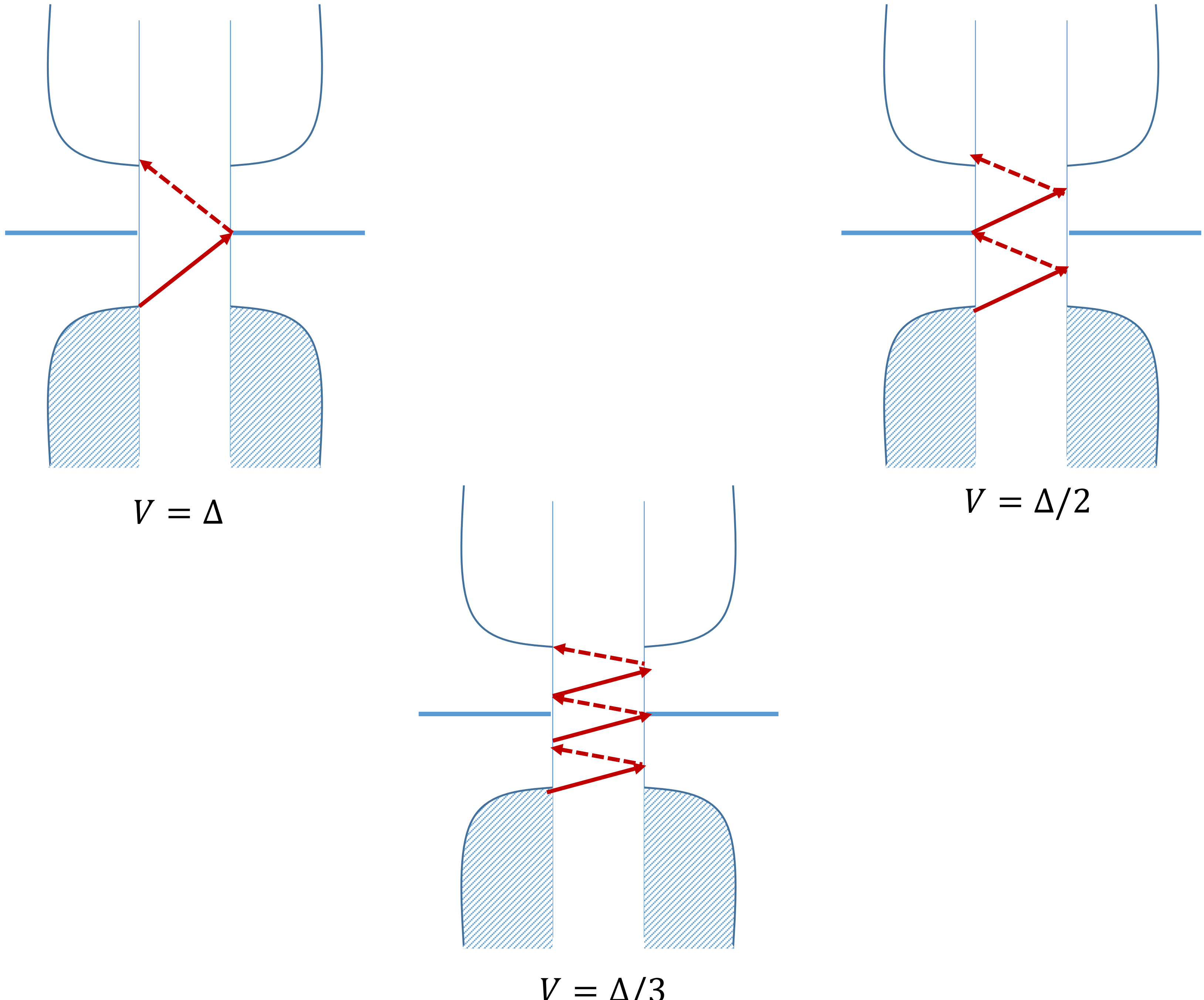
Non-equilibrium TS-TS case: MAR regime

Zazunov, Egger & ALY, PRB (2016)



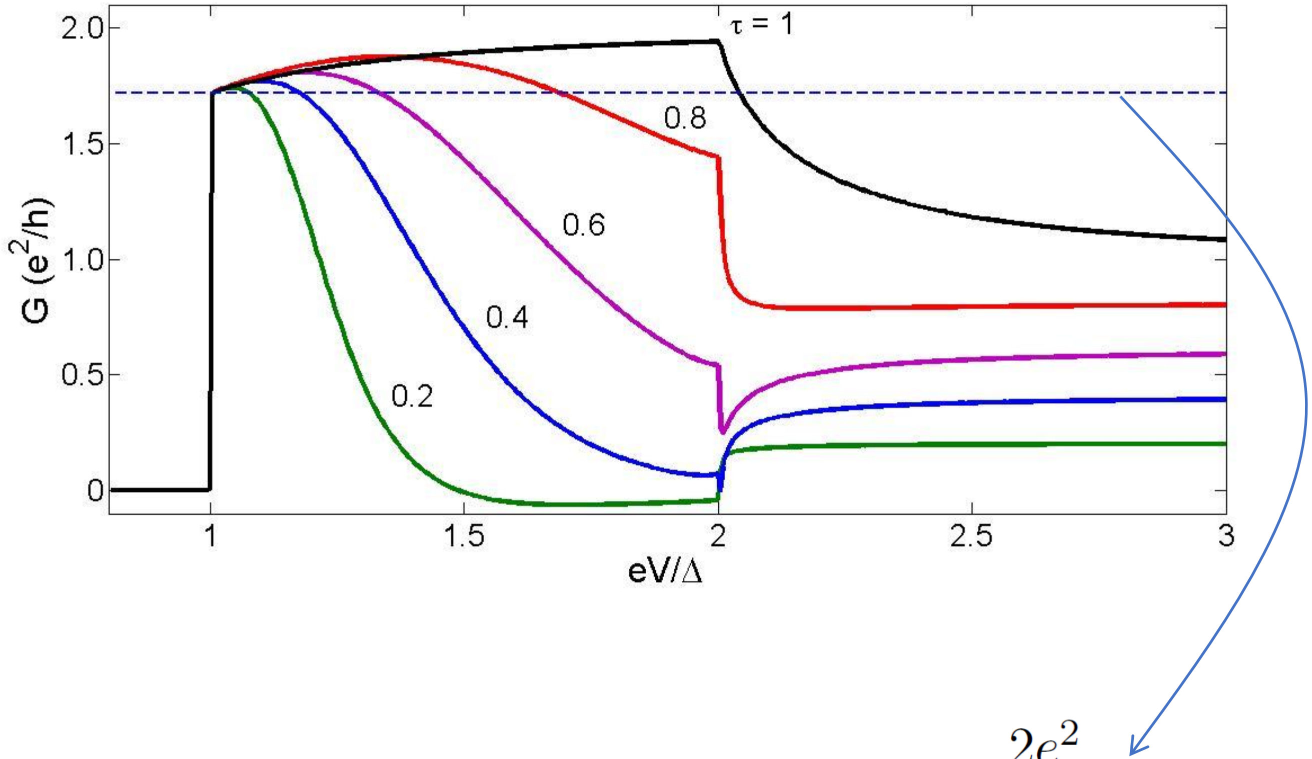
Subgap features at Δ/n instead of 2Δ/n

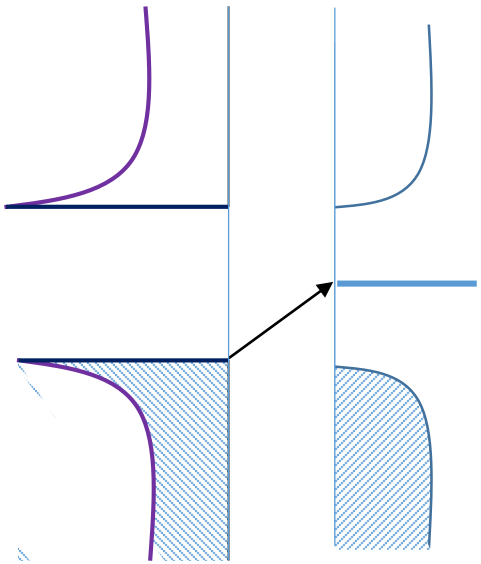
Badiane et al., PRL (2011)
San José et al., NJP (2013)


$V = \Delta$
$V = \Delta/2$
$V = \Delta/3$

S-TS case: differential conductance

Zazunov, Egger & ALY, PRB (2016)





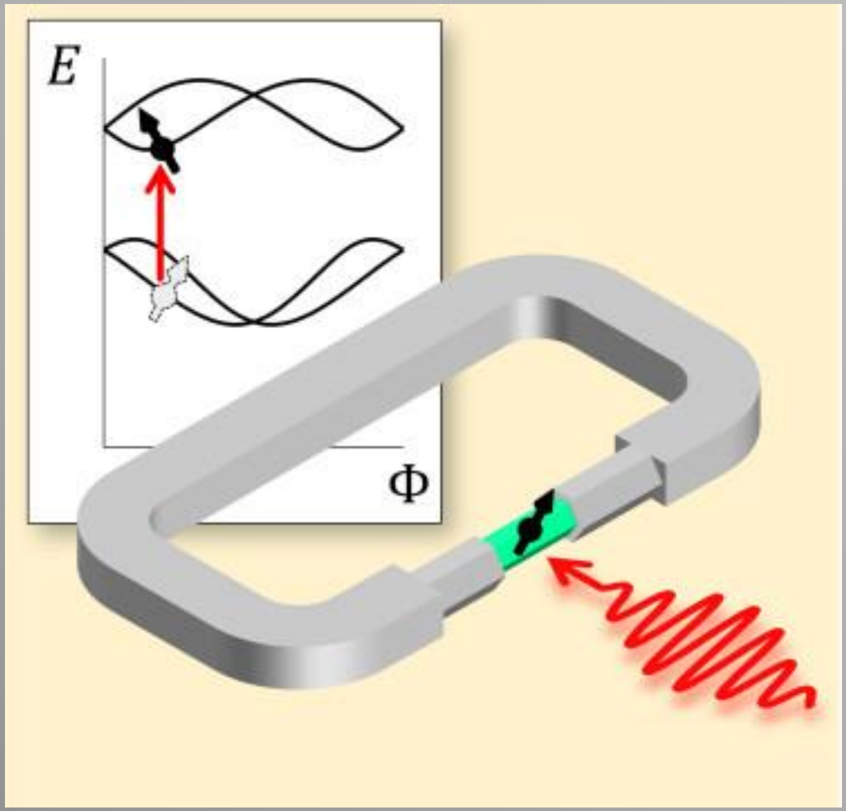
$V = \Delta$

$$G = (4 - \pi)\,\frac{2e^2}{h}$$

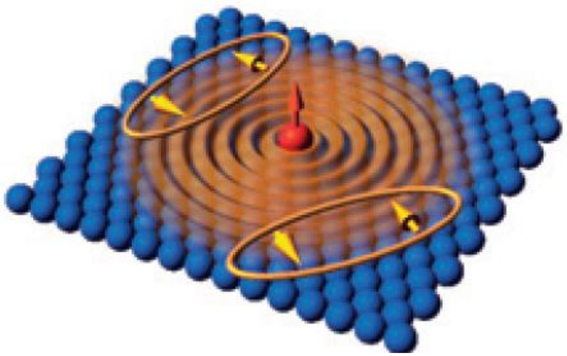
Peng et al., PRL (2015)

Behavior for spinful model → Setiawan et al., PRB (2017)

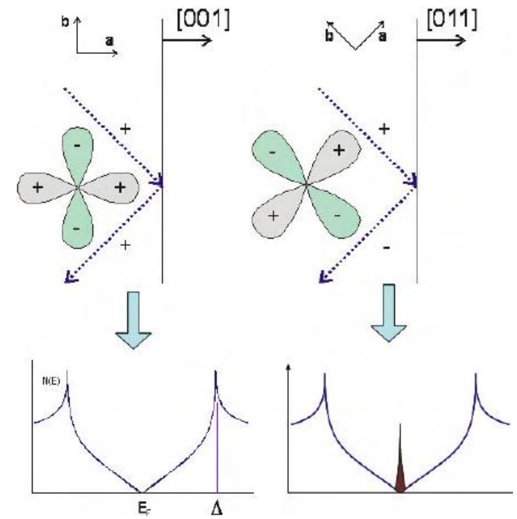
Second part: Detection and manipulation of Andreev states in hybrid nanowire Josephson junctions



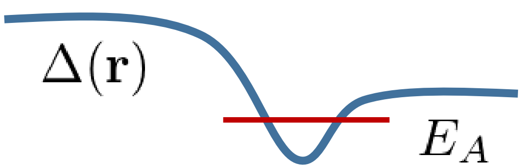
Shiba states

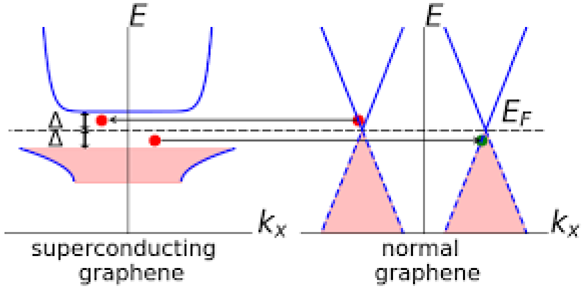


Andreev surface states



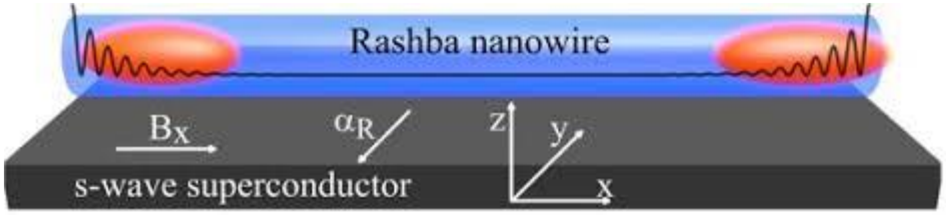
Andreev states



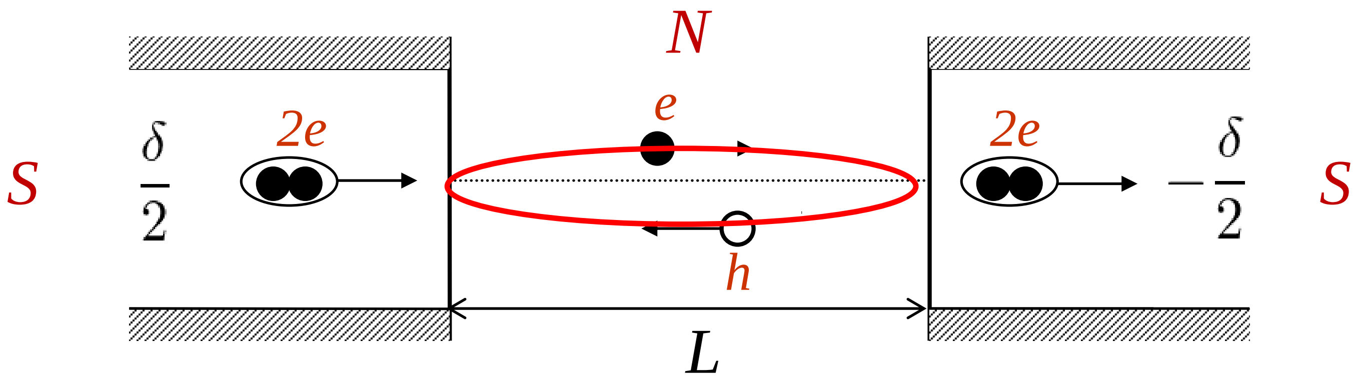


Interface bound states

Majorana bound states



Andreev states and Josephson effect

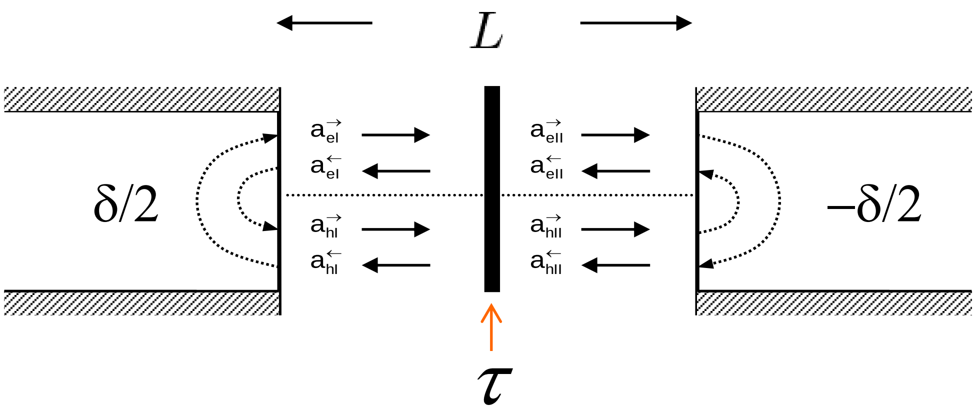


Condition for perfect transparency: $\delta - 2\arccos\left(\frac{E}{\Delta}\right) - (k_e - k_h)\,L = 2n\pi$

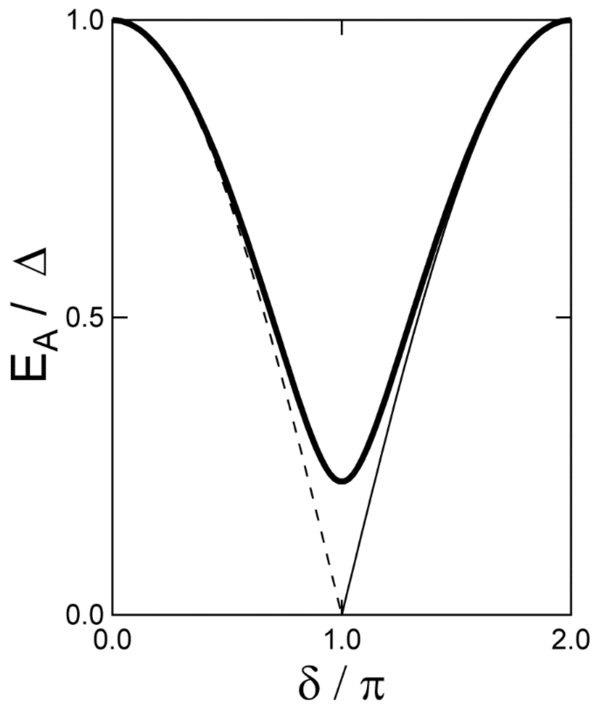
Kulik 70's

$E_n(\delta)$ ***current-carrying states***

ABSs for a short single channel + scatterer



$$L/\xi \ll 1 \longrightarrow E_A = \Delta\sqrt{1 - \tau \sin^2\frac{\delta}{2}}$$

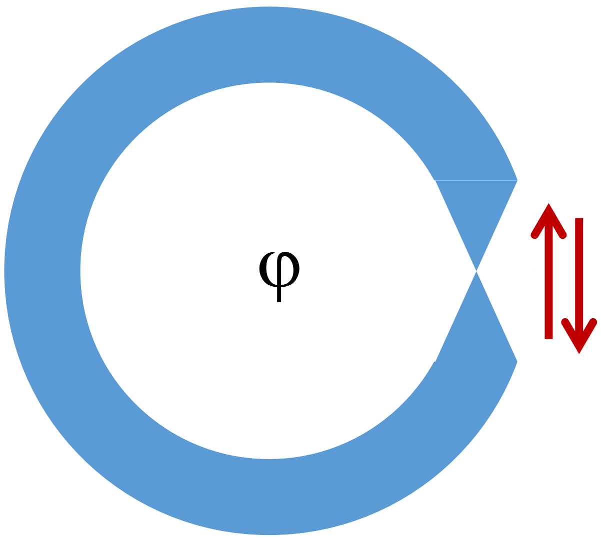


gs supercurrent

Bagwell, Beenaker, etc 90's

$$I = -\frac{e}{\hbar}\frac{\partial E}{\partial \delta} = \frac{e\Delta}{2\hbar}\frac{\tau \sin\delta}{\sqrt{1 - \tau \sin^2\frac{\delta}{2}}}$$

Andreev level qubit (ALQ)



$$H_{\text{eff}} = \Delta e^{-ir\sigma_x \delta/2}\left\{\cos\frac{\delta}{2}\sigma_z + r\sin\frac{\delta}{2}\sigma_y\right\}$$

$$r = \sqrt{1-\tau}$$

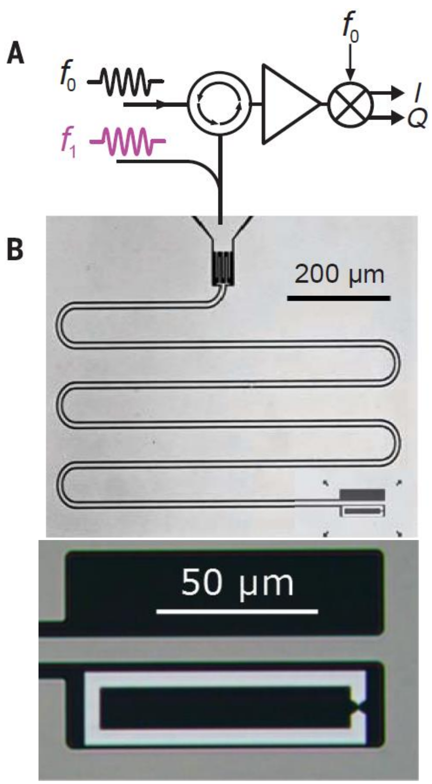
Superconducting flux qubit
Based on single quasiparticle excitations (instead of collective ones)

AndQC

https://www.andqc.eu/

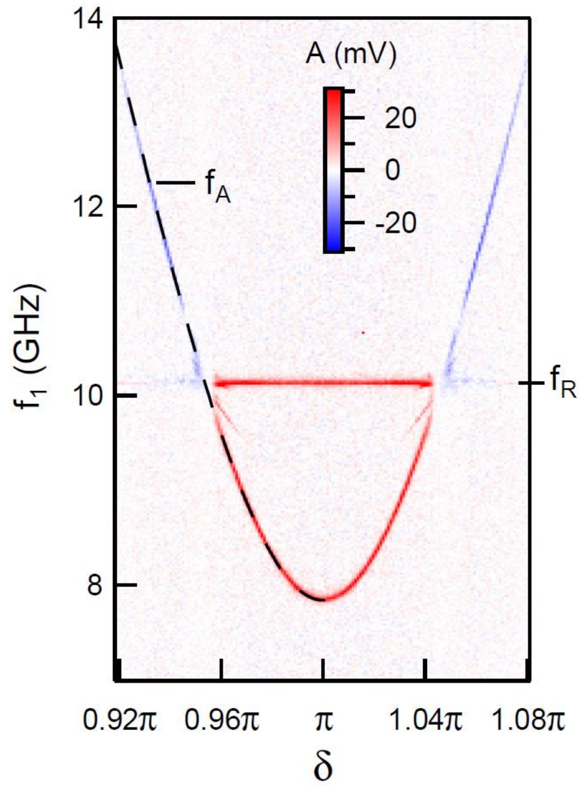
*M. Despósito and ALY, PRB **64**, 140511 (2001)*
*A. Zazunov et al. PRL **90**, 087003 (2003)*

Microwave detection and manipulation techniques

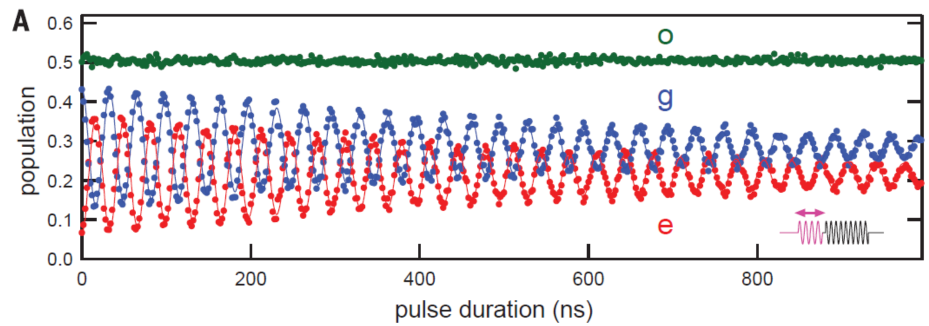


cQED techniques

Janvier et al., Science (2015)

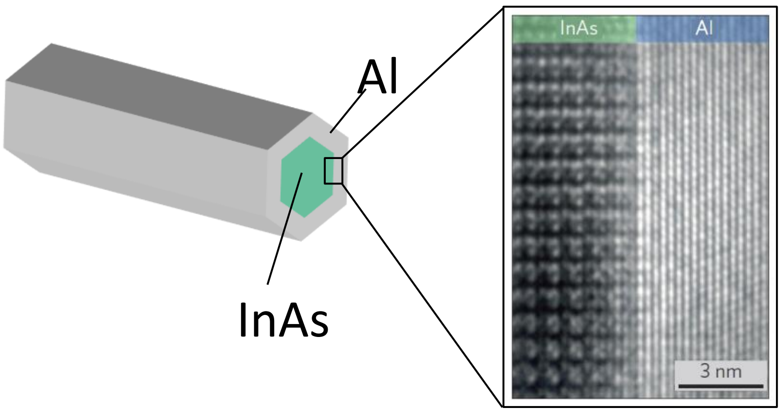


Two-tone spectroscopy



Time resolved measurements

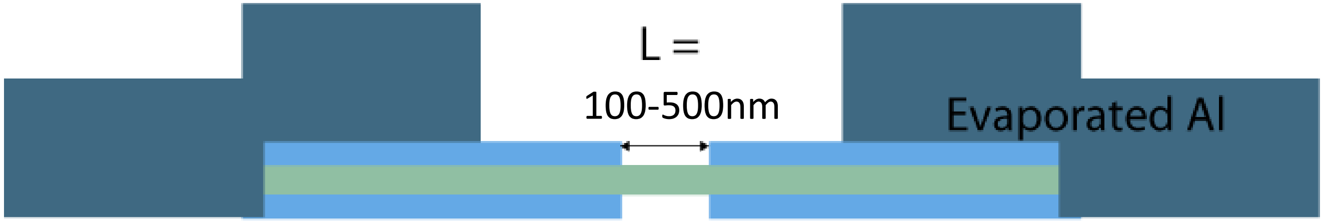
Josephson junctions based on semiconducting nanowires



Strong Rashba spin-orbit coupling

Copenhagen group

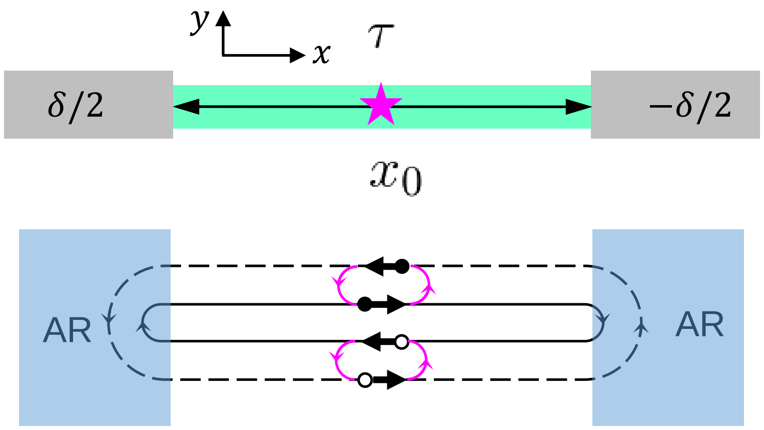
P. Krogstrup, J. Nygard, etc



Epitaxial Al/InAs nanowire

Effect of finite length? **Effect of spin-orbit?** **Effect of interactions?**

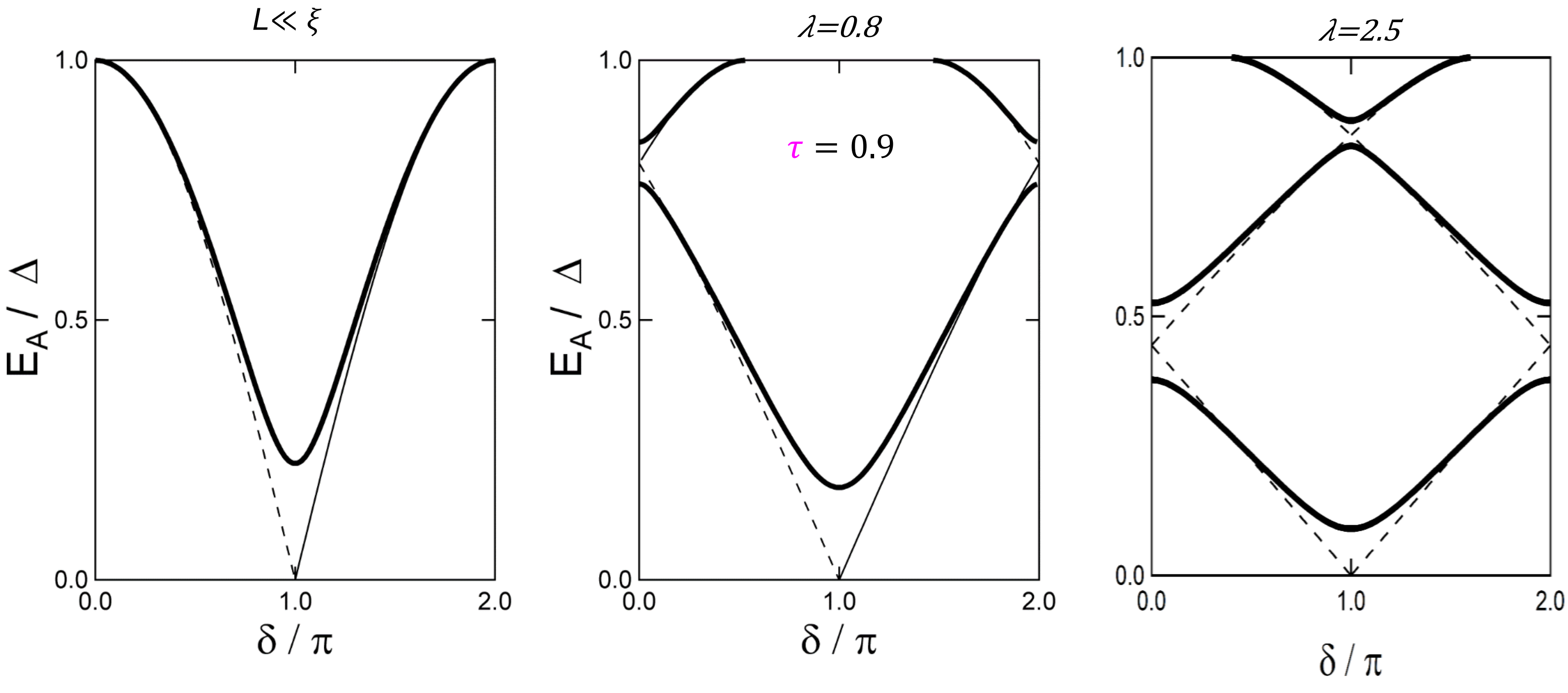
Effect of length: ABSs for a single channel + scatterer



$$x_r = \frac{2x_0}{L} \quad \varepsilon = \frac{E}{\Delta} \quad \lambda = \frac{L}{\xi}$$

Bagwell, PRB 1992

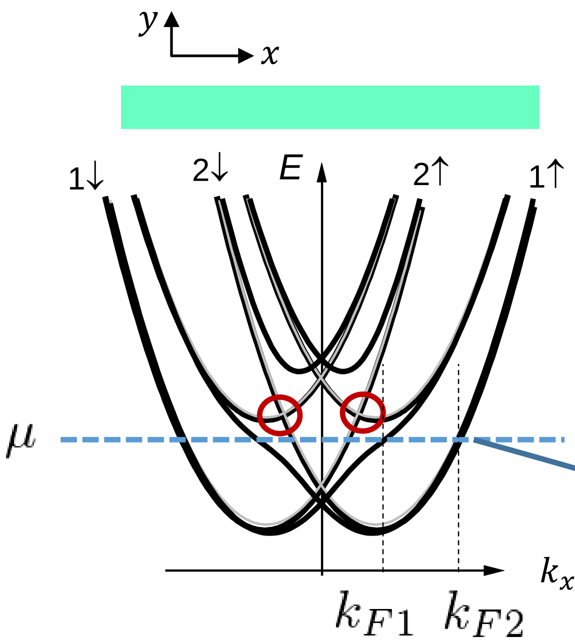
$$\tau\cos(\delta) + (1-\tau)\cos(2\lambda\varepsilon x_r) = \cos(2\arccos\varepsilon - 2\lambda\varepsilon)$$



Effect of Rashba SOI in multichannel nanowire

Moroz and. Barnes, PRB (1999)

A. Reynoso et al., PRL (2008)

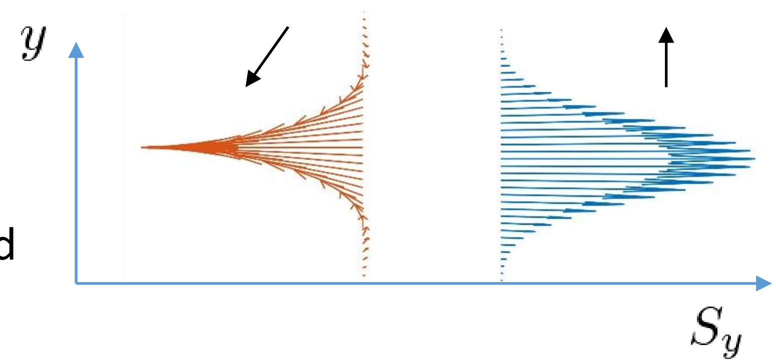


$$H = \frac{p^2}{2m^*} - \alpha\left(p_x\sigma_y - p_y\sigma_x\right)$$

$$= \frac{p_x^2}{2m^*} + \frac{p_y^2}{2m^*} - \alpha p_x\sigma_y + \alpha p_y\sigma_x$$

quantized

$$v_{F1} < v_{F2}$$

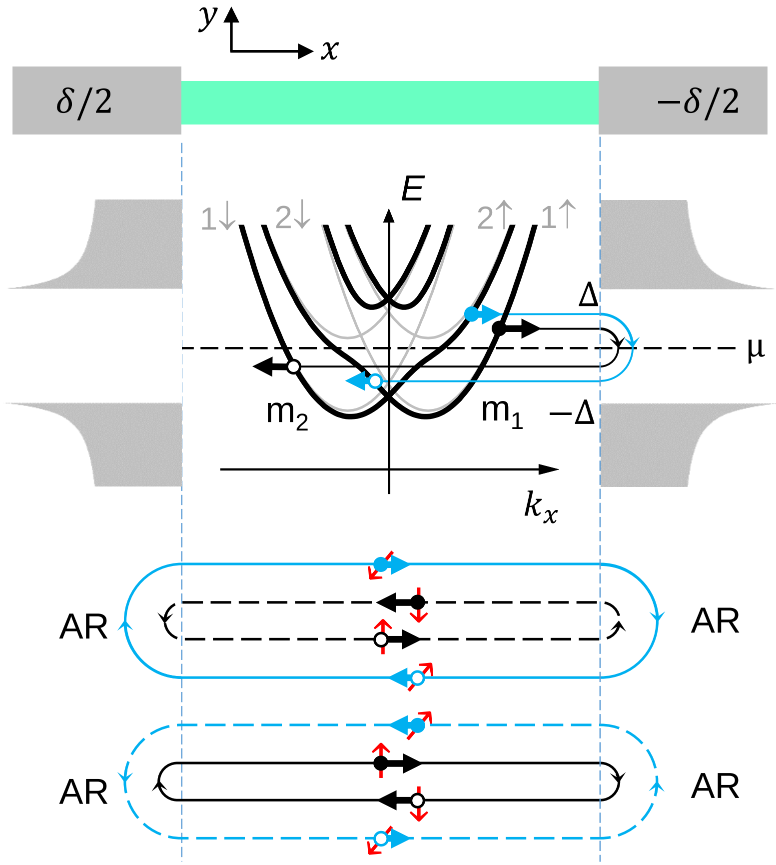


+ Spin texture in y direction

But pseudospin $\tau\sigma$ well defined

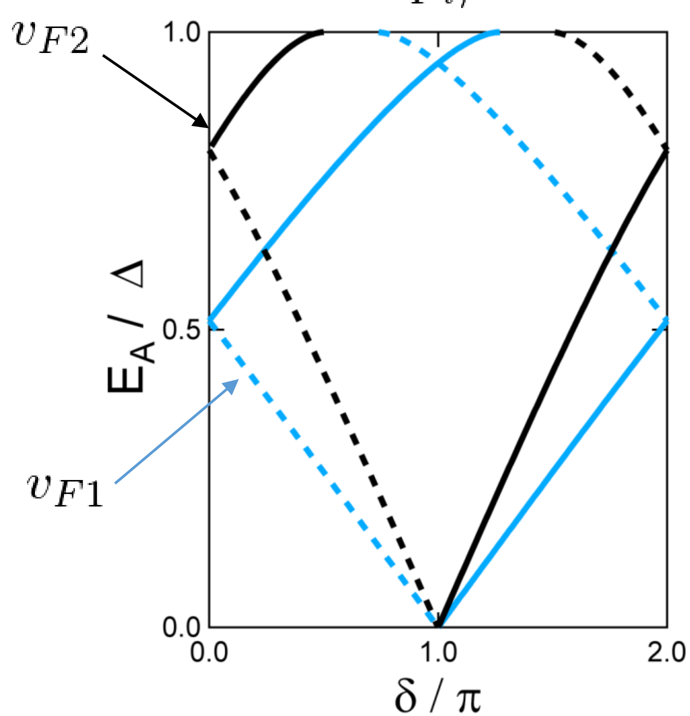
subband spin

Effect of Rashba SOI in multichannel nanowire



$$\pm\delta - 2\arccos\varepsilon - 2\varepsilon\lambda_i = 2n\pi$$

$$\lambda_i = \frac{L}{\hbar v_{Fi}/\Delta}$$



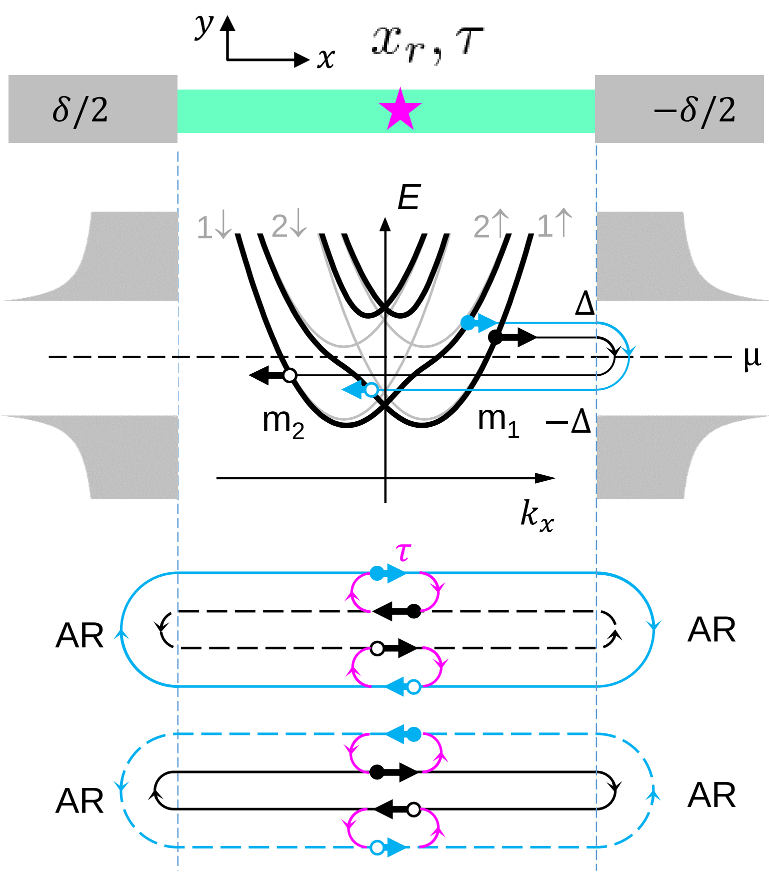
$$\lambda_2 = 2\lambda_1 = 2$$

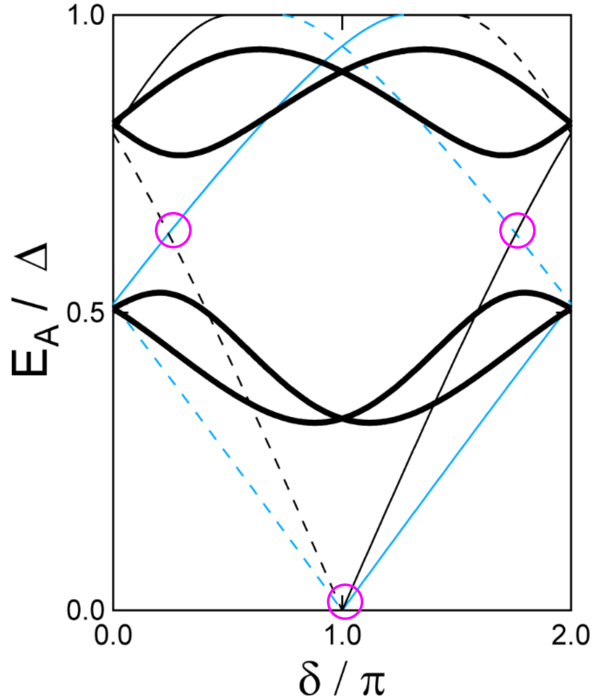
Park and ALY., PRB (2017)

Effect of Rashba SOI in multichannel nanowire

$$\lambda_2 = 2\lambda_1 = 2$$

$$\tau = x_r = 0.5$$

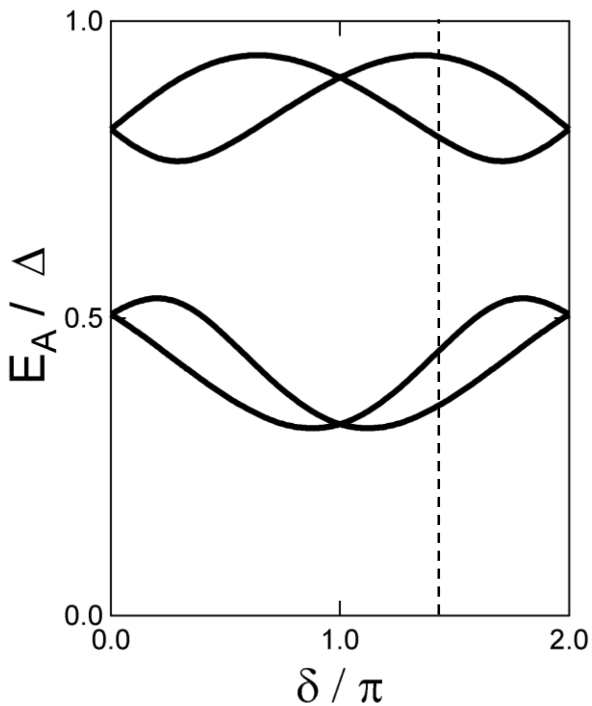




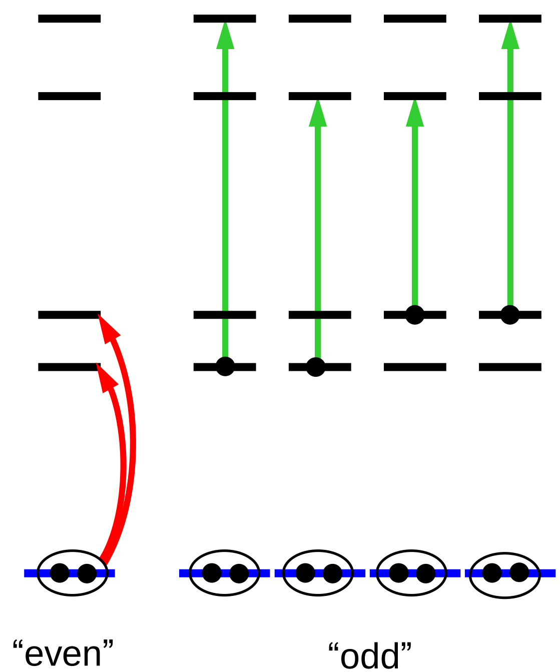
Tosi et al., PRX (2019)

$$\tau \cos\left((\lambda_1 - \lambda_2)\varepsilon \pm \delta\right) + (1-\tau)\cos\left((\lambda_1+\lambda_2)\varepsilon x_r\right) = \cos\left(2\arccos\varepsilon - (\lambda_1+\lambda_2)\varepsilon\right)$$

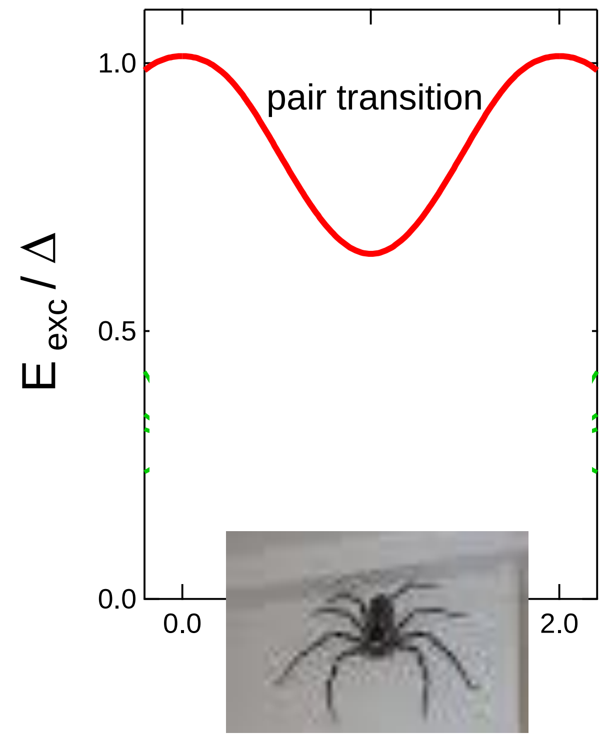
From ABSs to absorption spectrum



Andreev bound states



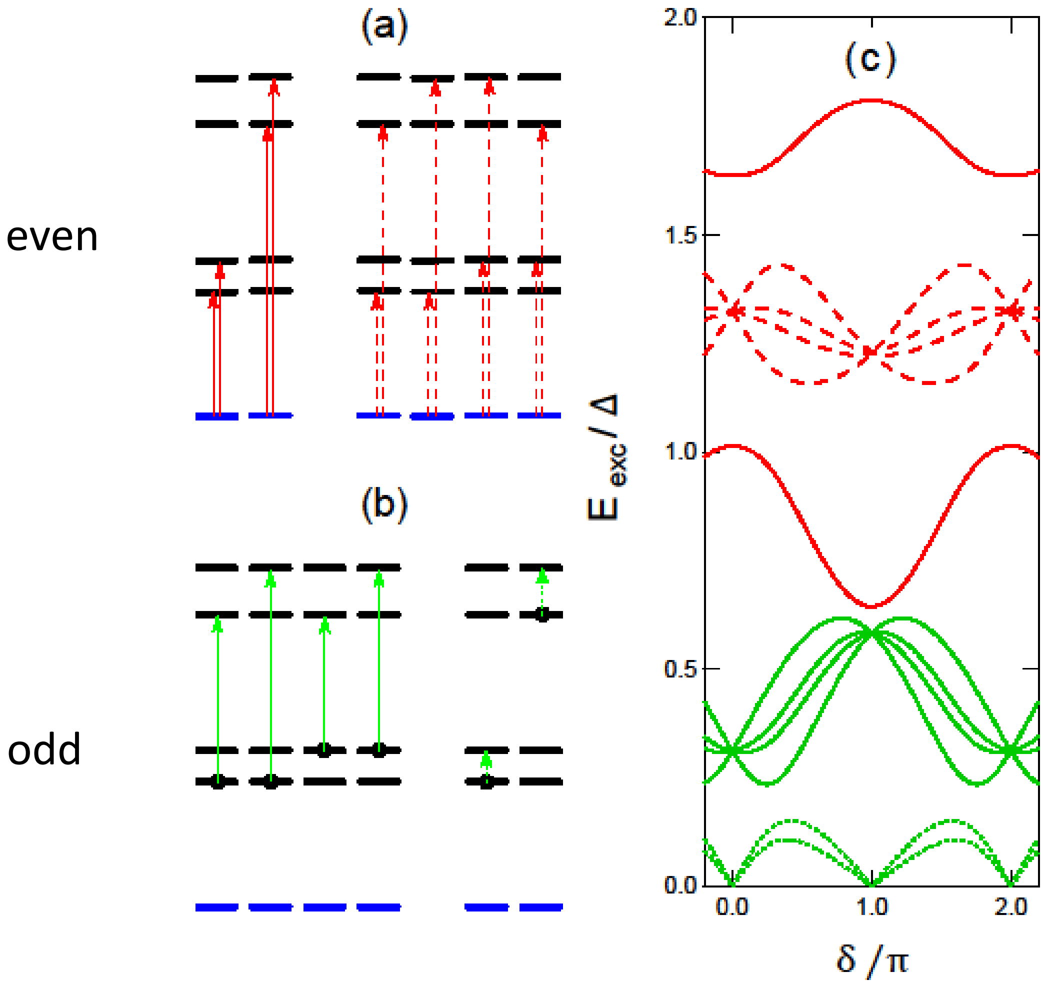
Transitions at a given phase



Absorption spectrum

Tosi et al., PRX (2019)

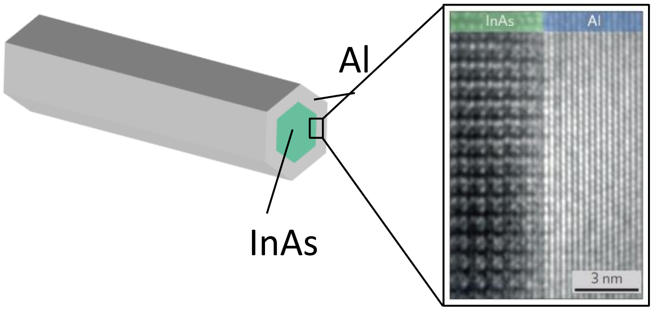
From ABSs to absorption spectrum: more general

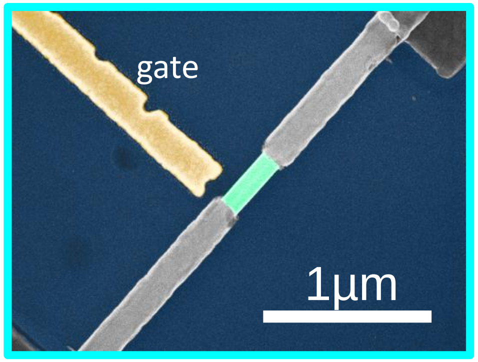


Tosi et al., PRX (2019)

Comparison to experiments

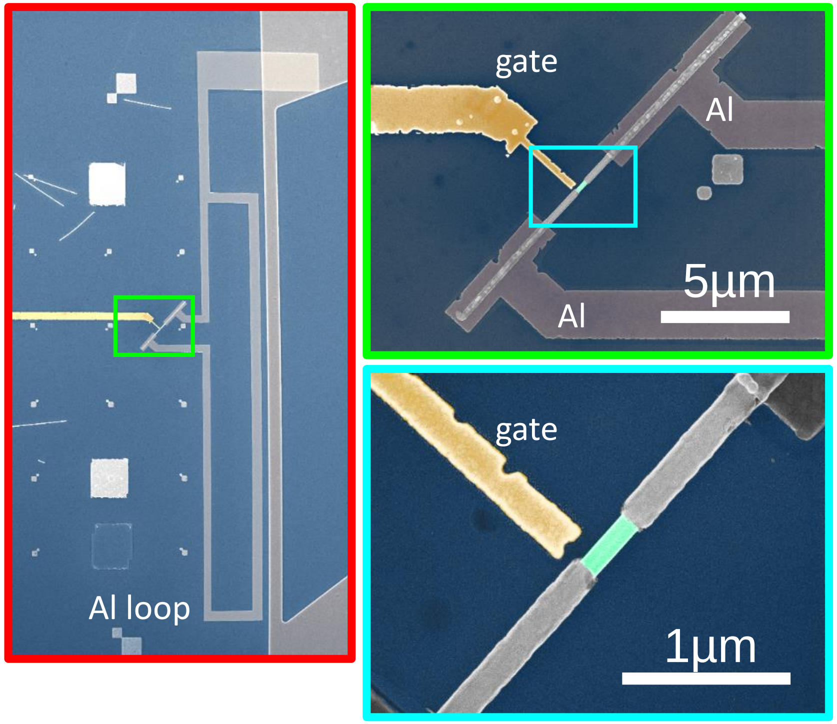
The Saclay experiment: InAs/Al core-shell nanowires



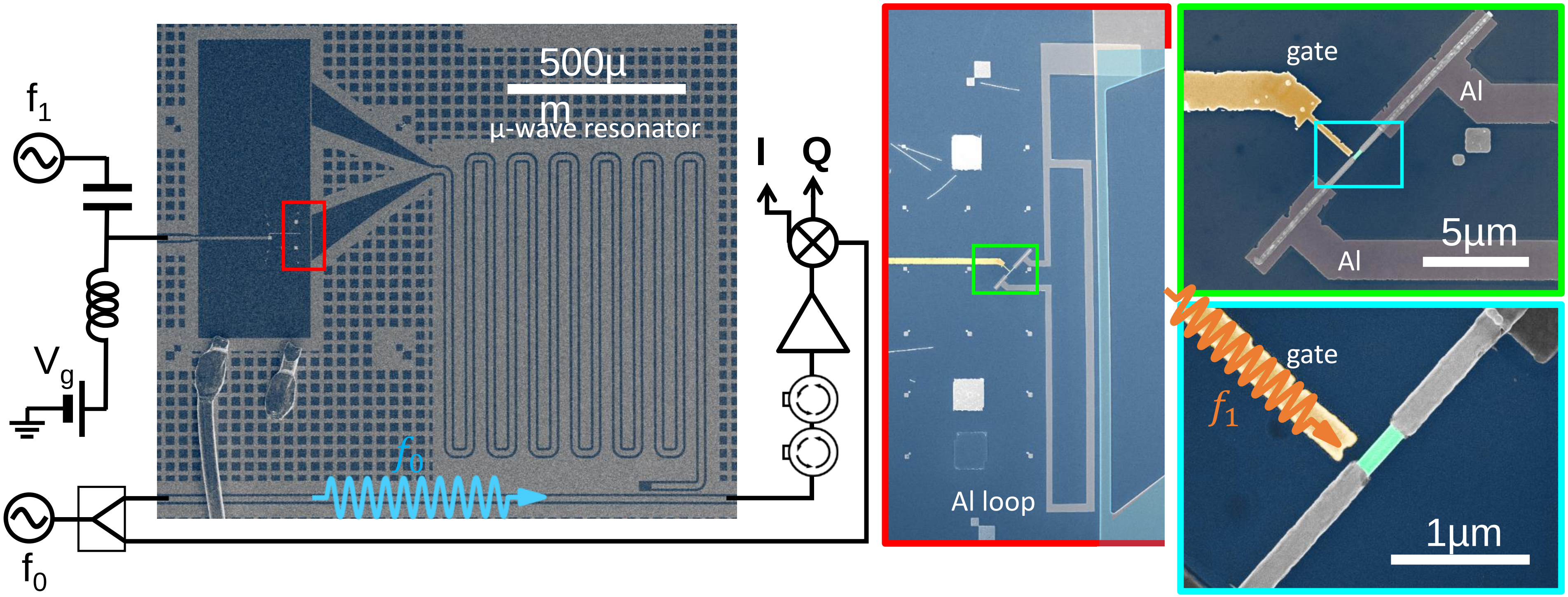


Tosi et al., PRX (2019)

The Saclay experiment: c-QED detection technique

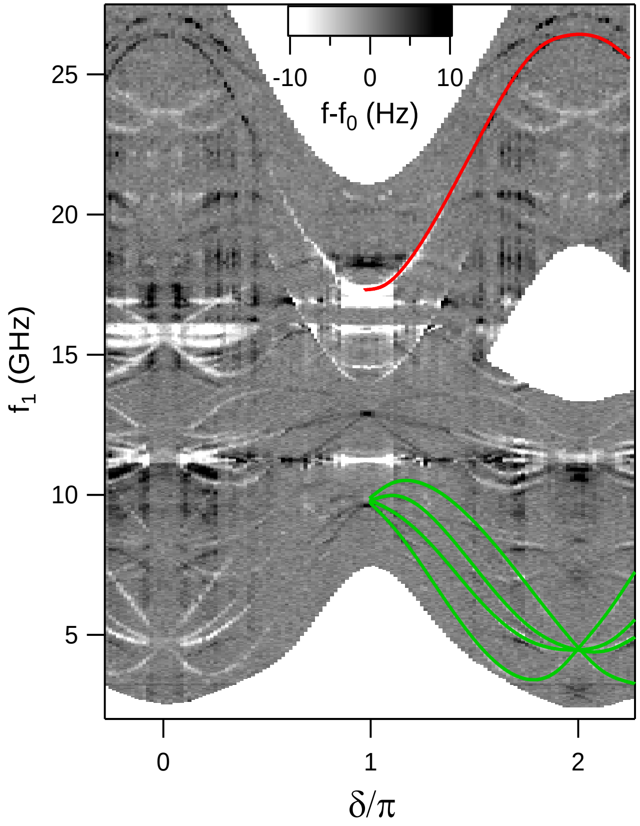


The Saclay experiment: c-QED detection technique

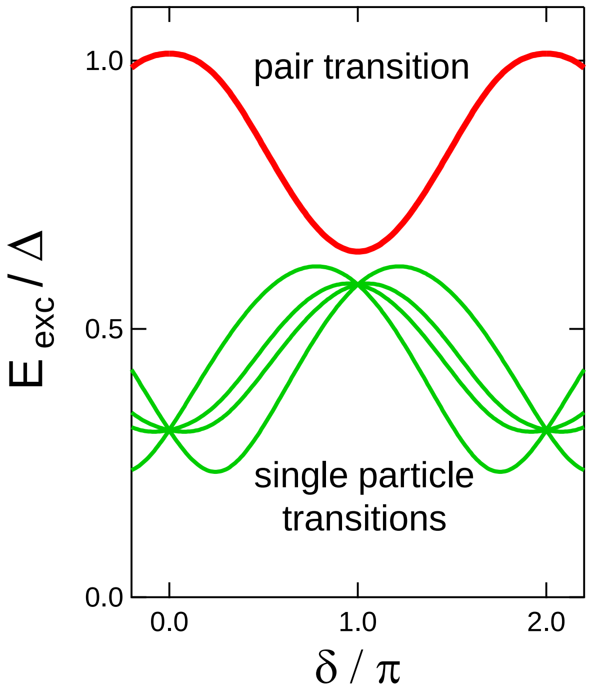


$$f_0 = f_r \simeq 3.26\text{GHz} \qquad Q_{int} \simeq 3 \times 10^5$$

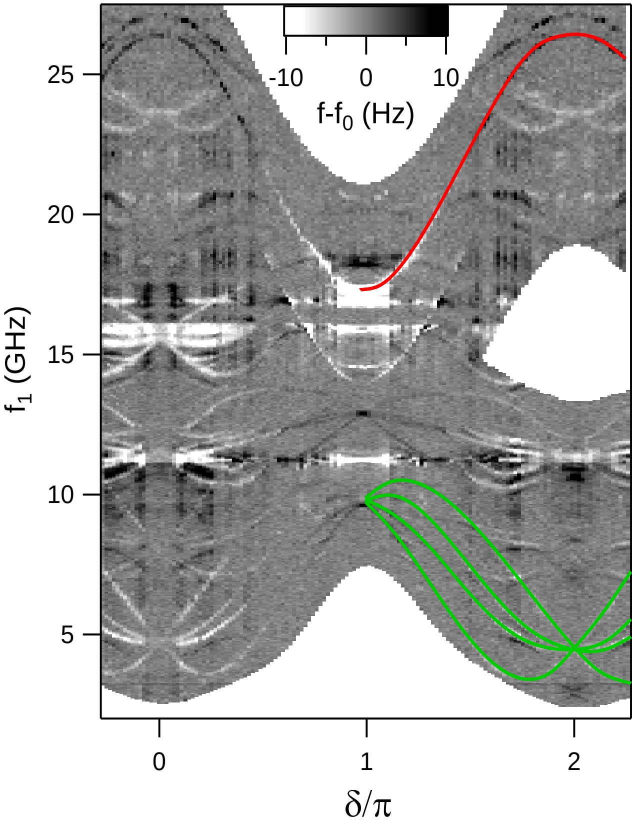
The Saclay experiment: absorption spectra



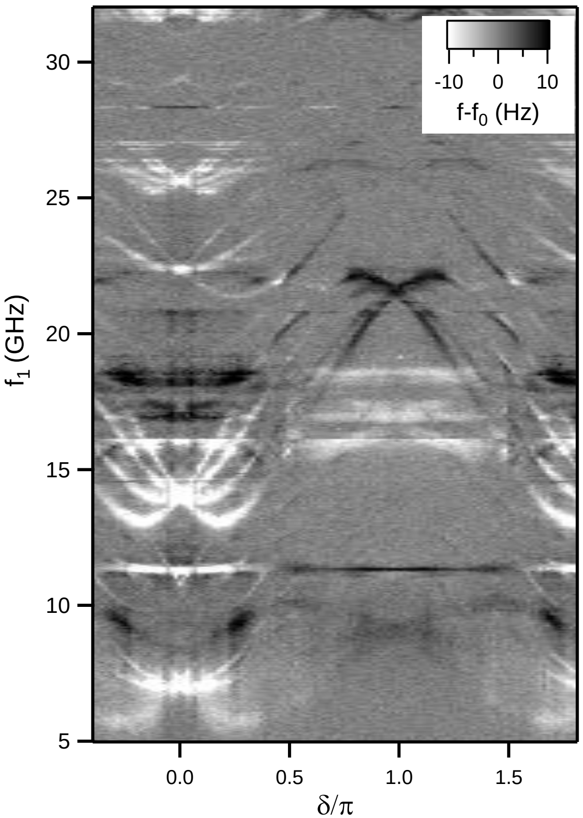
V_G=-0.89V



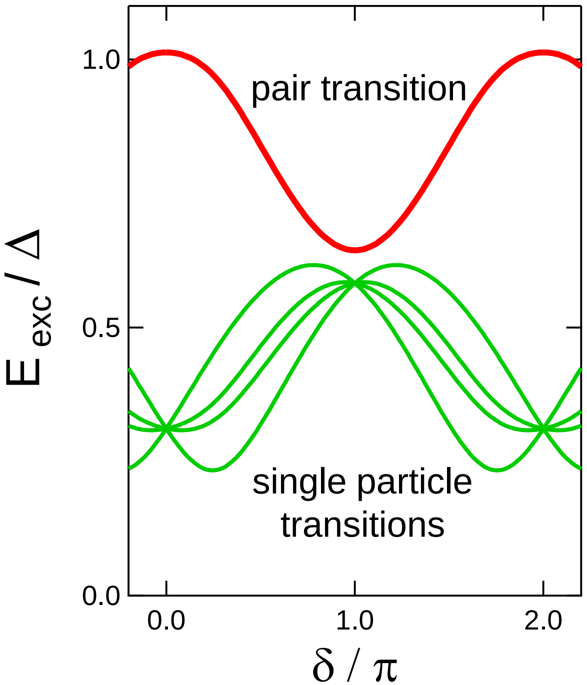
The Saclay experiment: absorption spectra



V_G=-0.89V



V_G=0.5V

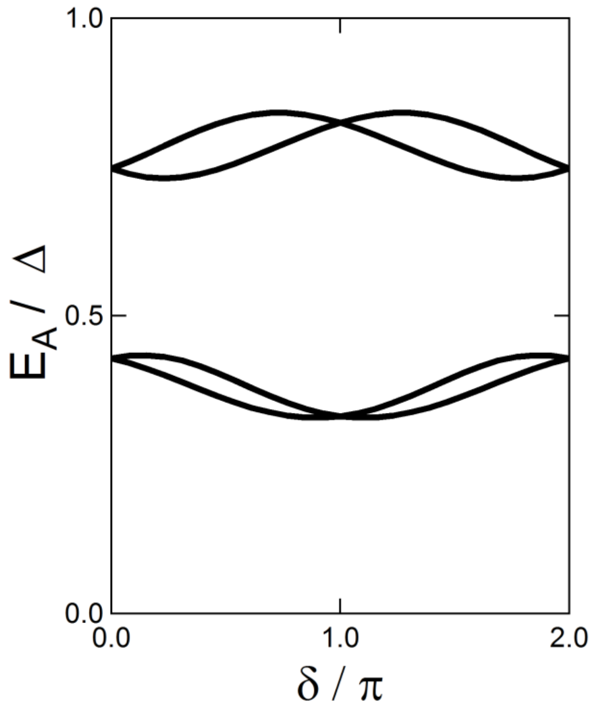


Fit with theory

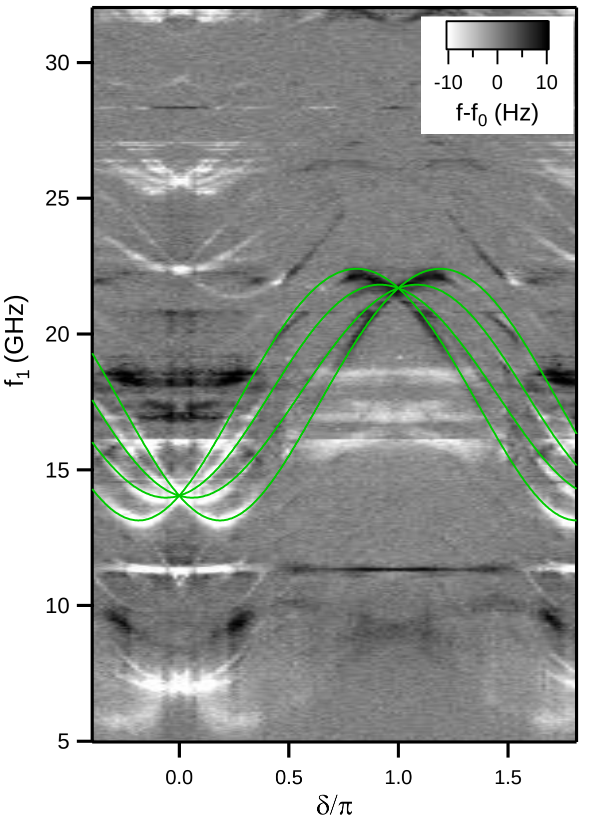
$$\tau\cos\left((\lambda_1-\lambda_2)\varepsilon\pm\delta\right)+(1-\tau)\cos\left((\lambda_1+\lambda_2)\varepsilon x_r\right)=\cos\left(2\arccos\varepsilon-(\lambda_1+\lambda_2)\varepsilon\right)$$

$\lambda_1 = 1.3$ $\quad \tau = 0.295$

$\lambda_2 = 2.3$ $\quad x_r = 0.525$

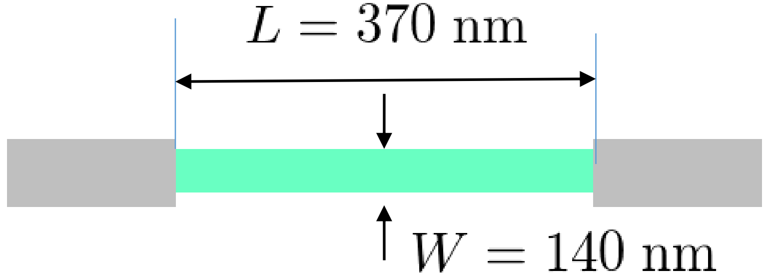


Andreev bound states



Absorption spectrum

Conversion into physical parameters

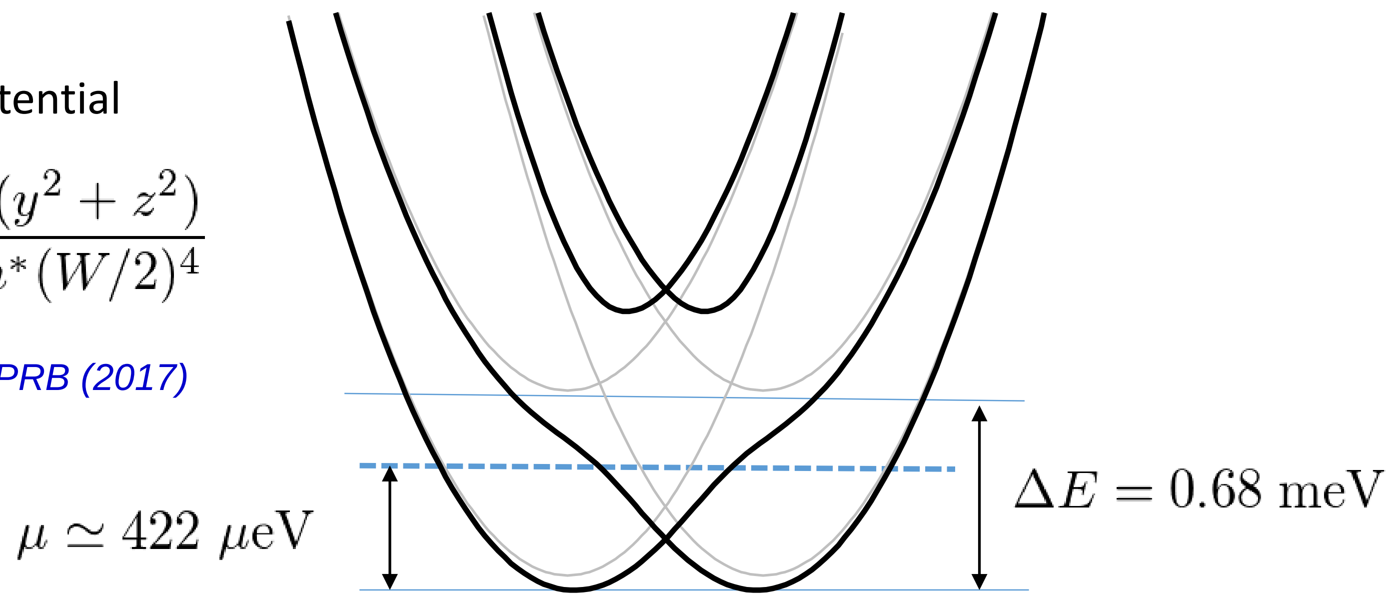


Warning: model dependent!

Confining potential

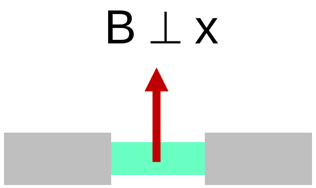
$$U(y,z) = \frac{\hbar^2(y^2+z^2)}{2m^*(W/2)^4}$$

Park and ALY., PRB (2017)



$\alpha \simeq 38$ meVnm

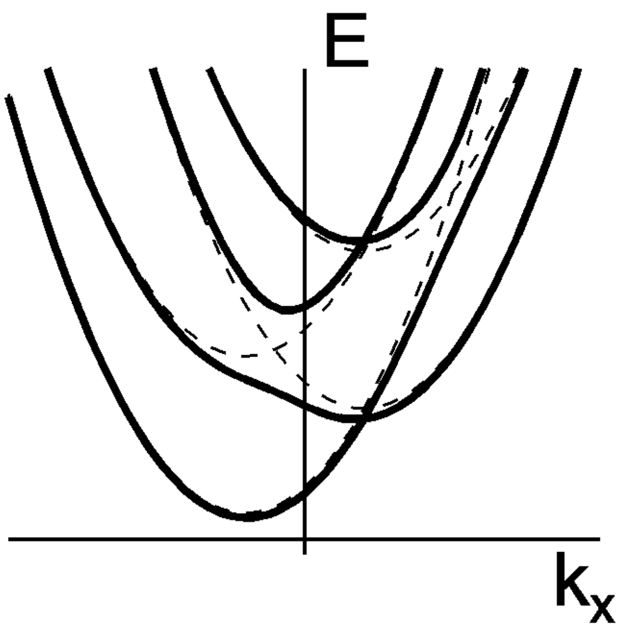
Including a magnetic field I: perp direction



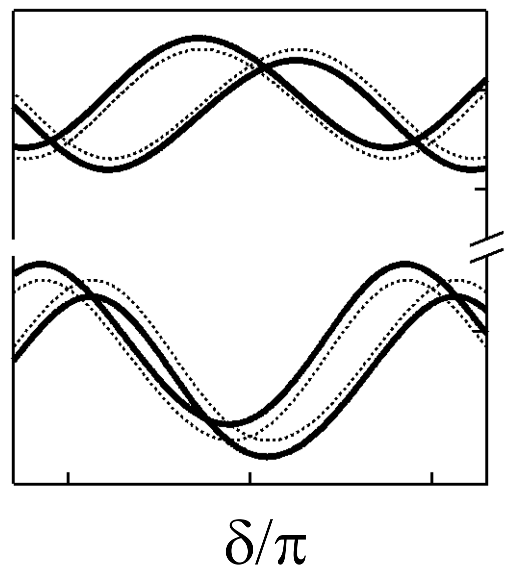
$$E_s(k_F) = \frac{\hbar^2 k_F^2}{2m^*} + \frac{E_1^\perp + E_2^\perp}{2} - \sqrt{\left[\frac{E_1^\perp - E_2^\perp}{2} - s\left(\alpha k_F - \frac{g_\perp \mu_B}{2} B_y\right)\right]^2 + \eta^2}$$

$$E_n^\perp = \frac{4\hbar^2 n}{m^* W^2} \qquad \eta = \frac{\sqrt{2}\alpha}{W}$$

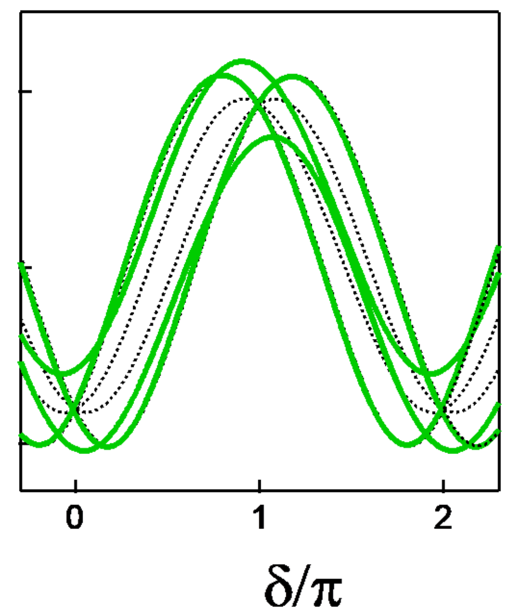
Park and ALY., PRB (2017)



Band dispersion

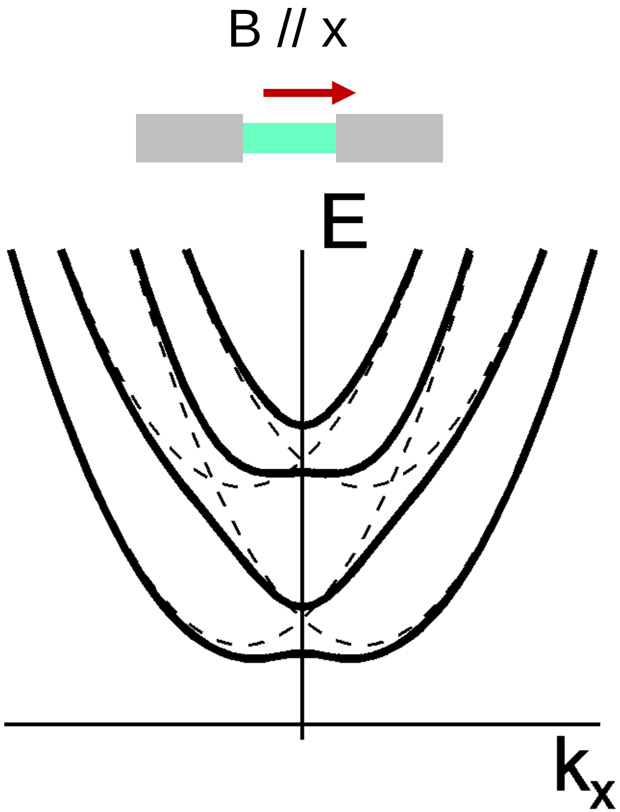


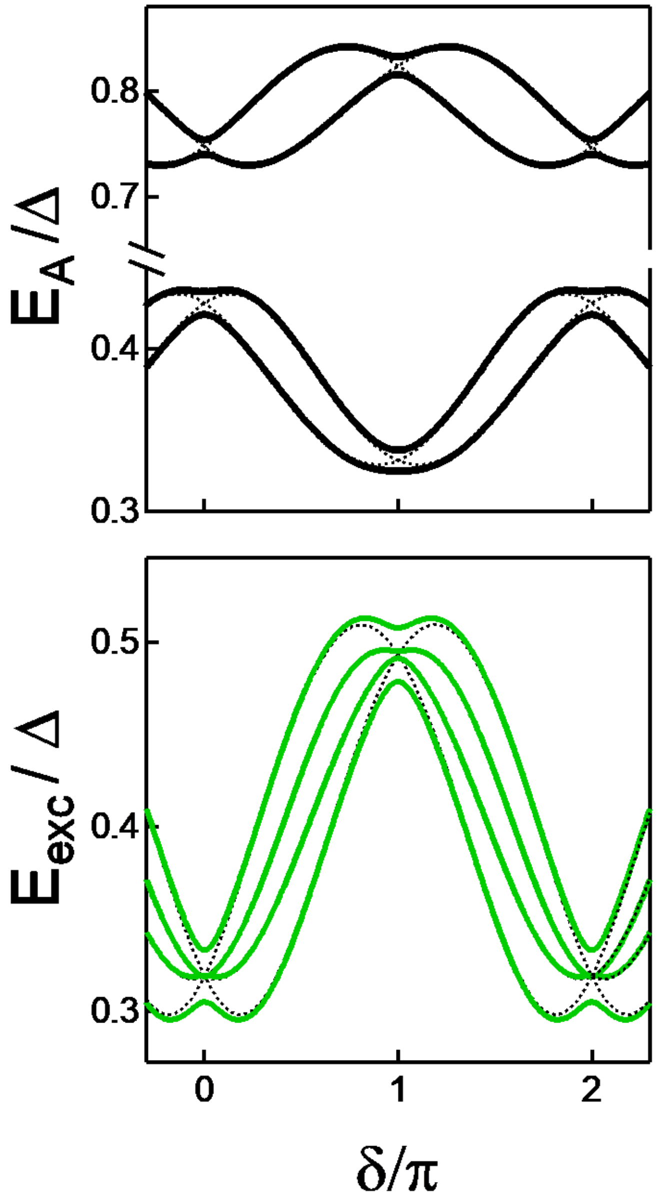
ABSs



single qp transitions

Including a magnetic field II: parallel direction

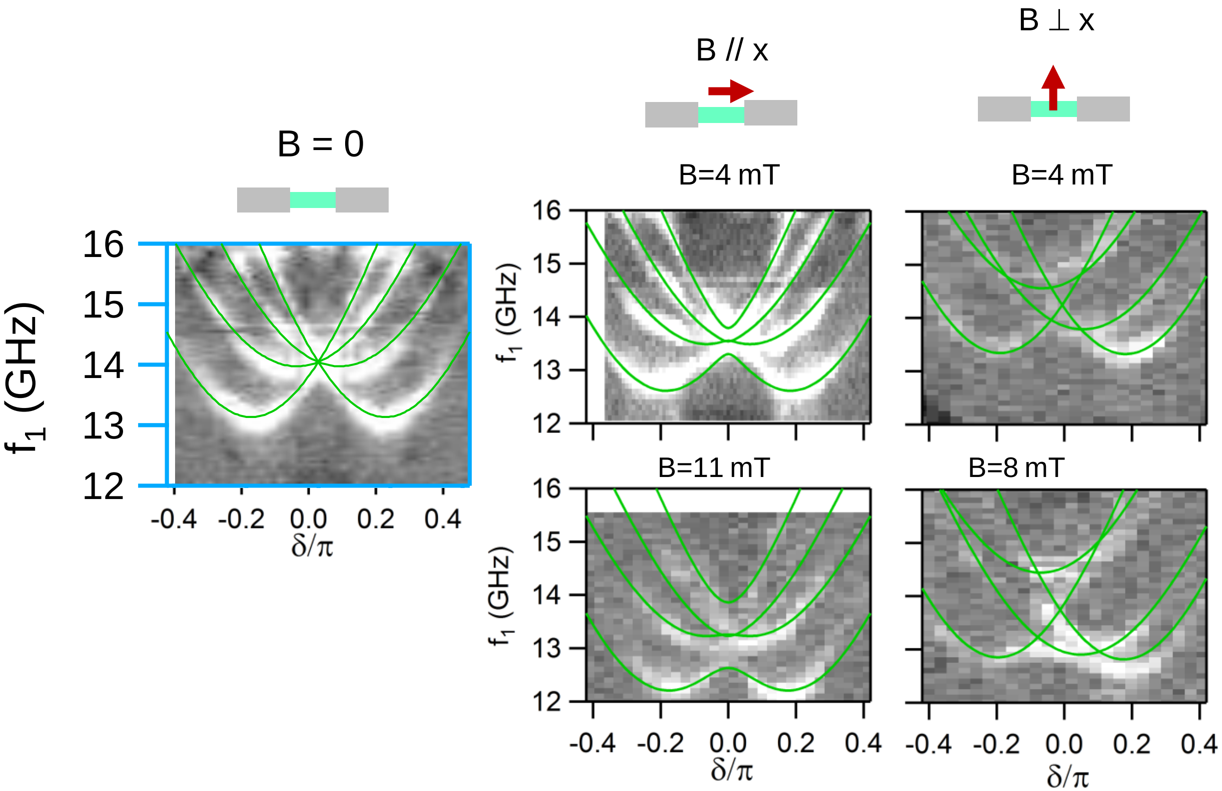




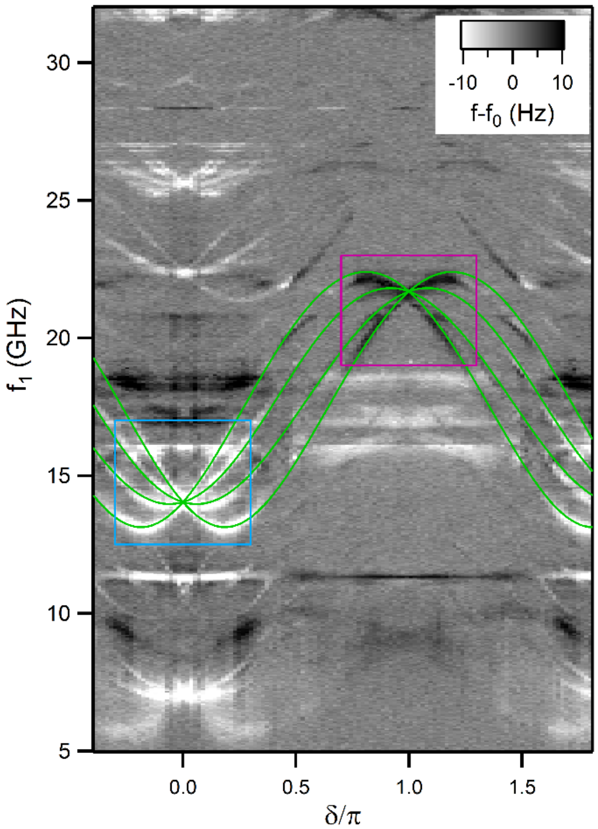
ABS

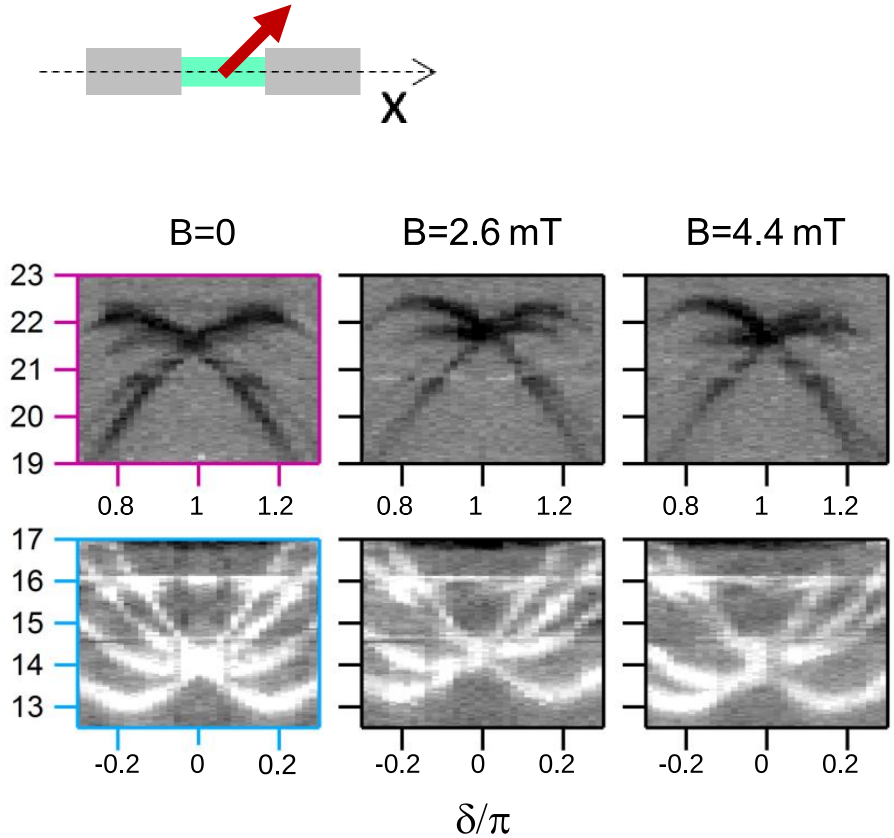
single qp transitions

Fit with theory including magnetic field



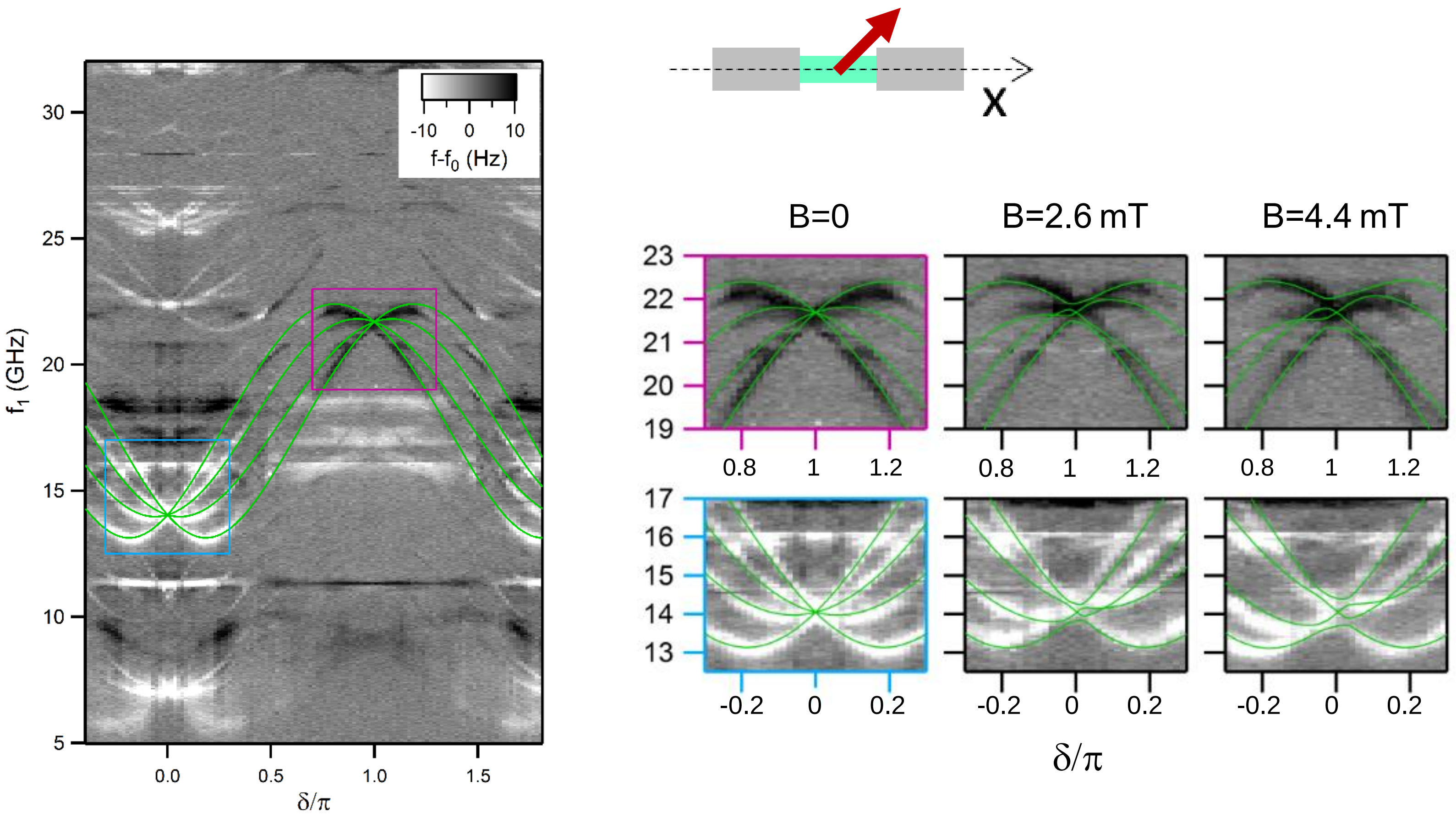
Fit with theory including magnetic field





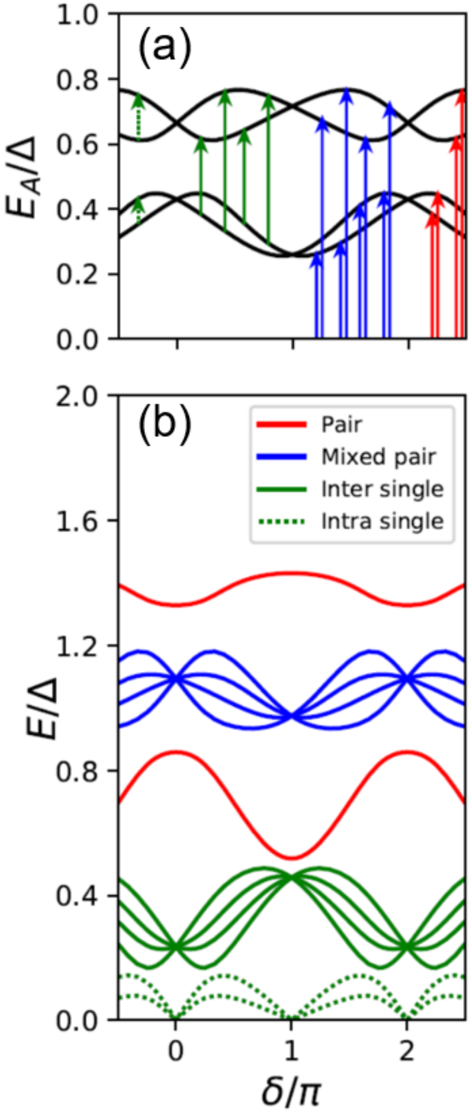
Tosi et al., PRX (2019)

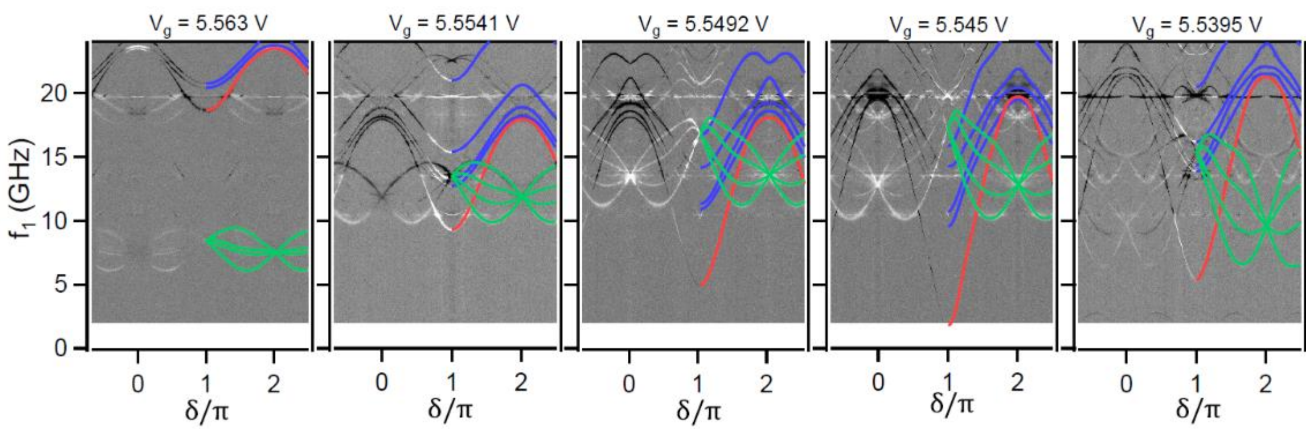
Fit with theory including magnetic field



Tosi et al., PRX (2019)

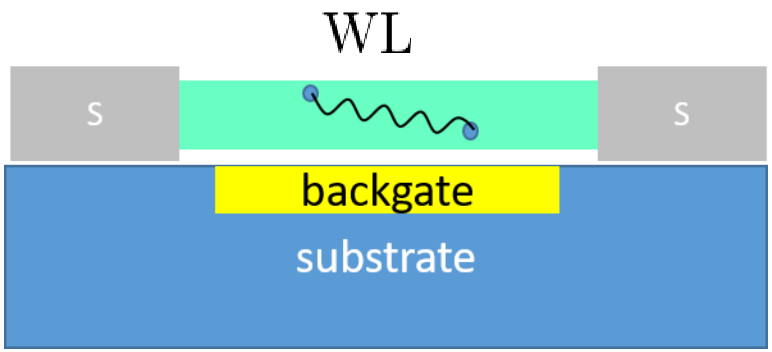
Other transitions?





Effect of electron-electron interactions

F.J. Matute Cañadas, C. Metzger et al. PRL (2022)

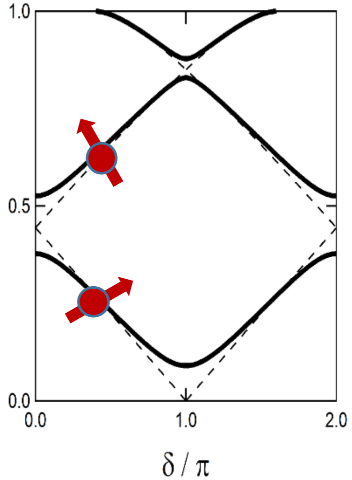


$$\hat{V} = \frac{1}{2}\sum_{\sigma,\sigma'}\int_{\rm WL} d\mathbf{r}d\mathbf{r}'\,\Psi^\dagger_\sigma(\mathbf{r})\Psi^\dagger_{\sigma'}(\mathbf{r}')u(\mathbf{r}-\mathbf{r}')\Psi_{\sigma'}(\mathbf{r}')\Psi_\sigma(\mathbf{r})$$

$$u(\mathbf{r}-\mathbf{r}') = u_0\delta(\mathbf{r}-\mathbf{r}')$$

Estimation (2D model) $u_0 \sim 3\ \mathrm{eVnm}^2$

$$E_c = u_0/A \sim 30\mu\mathrm{eV}$$



Effective exchange

$$-J\vec{S}^2, J \sim u_0/A \sim 5\mathrm{GHz}$$

$S = 0$

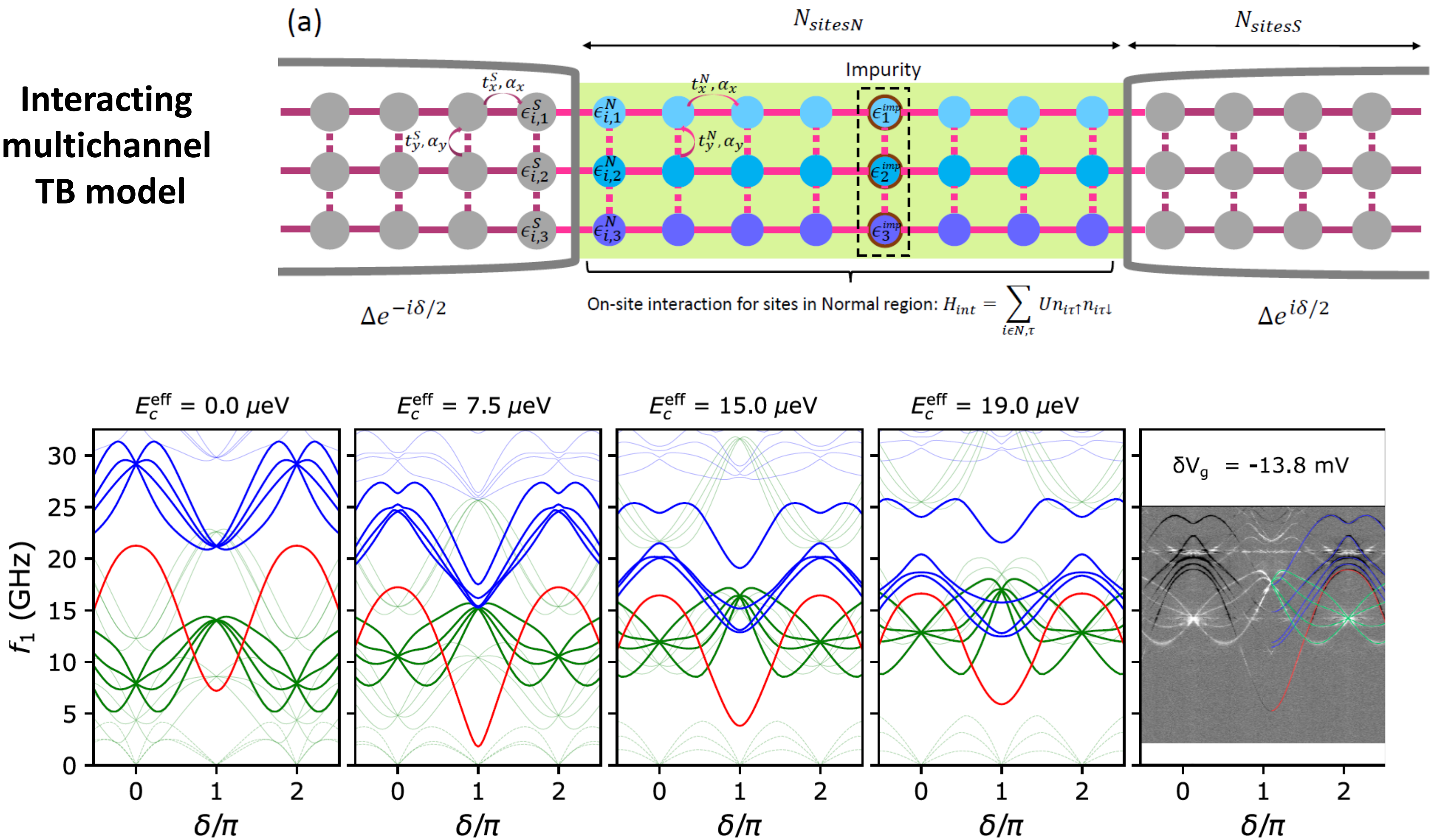
$S = 1$

$2J$

Effect of electron-electron interactions

F.J. Matute Cañadas, C. Metzger et al., PRL (2022)

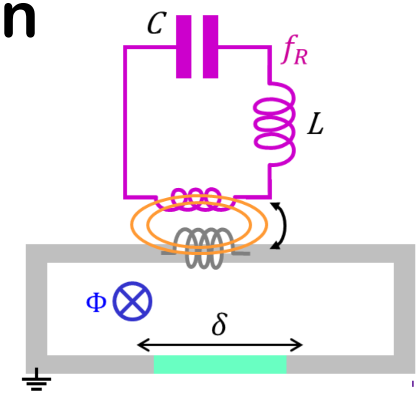
Interacting multichannel TB model



Related work: Fatemi et al. PRL (2022)

Towards ABSs+cQED theory

Theory of cQED detection



Park et al., PRL (2020)

(UAM-Saclay collaboration)

Nanowire   Resonator-NW coupling

$$H = H_0(\delta) + \lambda H_0'(\delta)\left(a + a^\dagger\right) + \frac{\lambda^2}{2} H_0''(\delta)\left(a + a^\dagger\right)^2 + \hbar\omega_R a^\dagger a$$

$$\{|\Phi_i n\rangle \equiv |\Phi_i\rangle \otimes |n\rangle\}$$ Uncoupled resonator-junction basis

$$\delta E_{i,n}^{(1)} = \frac{\lambda^2}{2}\langle\Phi_i n|H_0''\left(a + a^\dagger\right)^2|\Phi_i n\rangle = \frac{\lambda^2}{2}\langle\Phi_i|H_0''|\Phi_i\rangle(2n+1)$$

$$\delta E_{i,n}^{(2)} = -\lambda^2 \sum_{j\neq i} |\langle\Phi_j|H_0'|\Phi_i\rangle|^2 \left(\frac{n+1}{E_j + \omega_R - E_i} + \frac{n}{E_j - \omega_R - E_i}\right)$$

Theory of cQED detection

$$\delta E_{i,n} = \delta\omega_{R,i}\left(n+\frac{1}{2}\right) + \frac{\lambda^2}{2}\sum_{j\neq i}|\langle\Phi_j|H_0'|\Phi_i\rangle|^2\left(\frac{1}{E_j+\omega_R-E_i} - \frac{1}{E_j-\omega_R-E_i}\right)$$

$$\delta\omega_{R,i} = \lambda^2\left\{E_i'' - \sum_{j\neq i}|\langle\Phi_j|H_0'|\Phi_i\rangle|^2\left(\frac{1}{E_j+\omega_R-E_i} + \frac{1}{E_j-\omega_R-E_i} - \frac{2}{E_j-E_i}\right)\right\}$$

$\rightarrow 0 \text{ for } \omega_R \rightarrow 0$

Two regimes

$\min|E_i - E_j| \gg \omega_R \Rightarrow \delta\omega_{R,i} \propto E_i''$ Curvature

("adiabatic" regime)

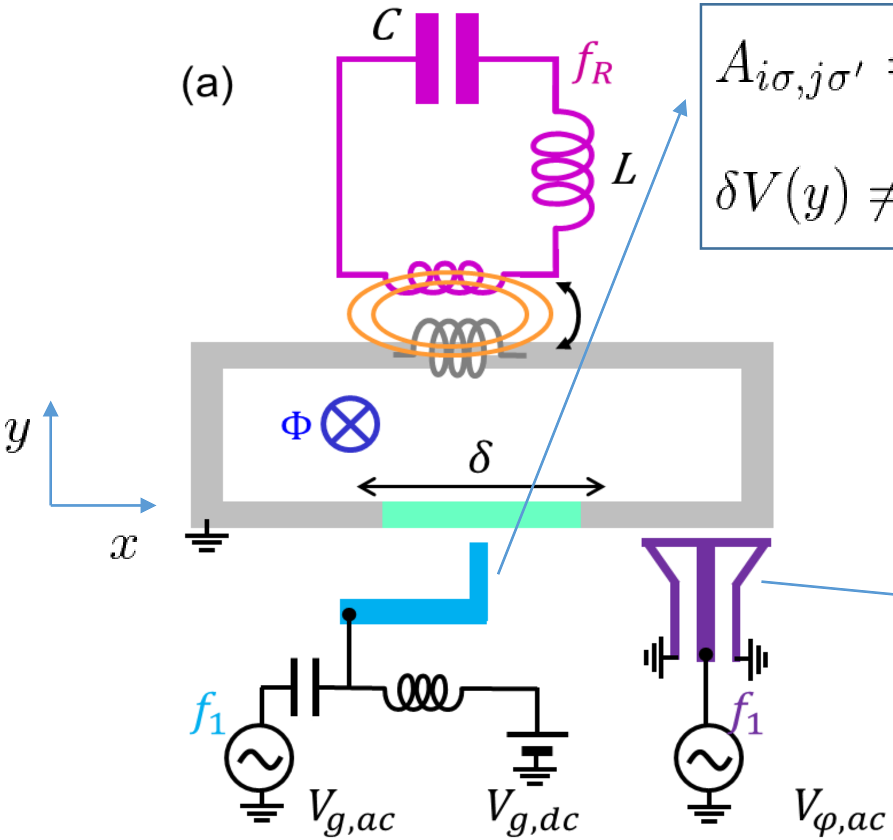
$\min|E_i - E_j| \simeq \omega_R \Rightarrow \delta\omega_{R,i} \propto \frac{|\langle\Phi_j|H_0'|\Phi_i\rangle|^2}{E_j-\omega_R-E_i}$ Jaynes-Cummings

("dispersive" regime)

From adiabatic to dispersive readout of quantum circuits *Park et al., PRL (2020)*

Theory of cQED manipulation: different driving fields

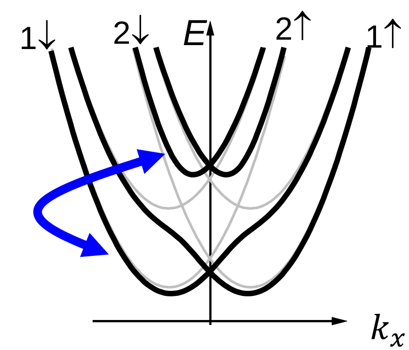
$$\hat{H}_d(t) = \frac{1}{2}\sum_{i\sigma<j\sigma'}\left(A_{i\sigma,j\sigma'}\gamma^{\dagger}_{i\sigma}\gamma_{j\sigma'}e^{i\omega_d t} + \text{h.c.}\right) \qquad \hat{H}_0 = \sum_{i\sigma} E_{i\sigma}(\delta)\gamma^{\dagger}_{i\sigma}\gamma_{i\sigma}$$



Gate drive

$$A_{i\sigma,j\sigma'} = \langle \Phi_{i\sigma}|\delta V(\vec{r})\tau_z|\Phi_{j\sigma'}\rangle$$

$$\delta V(y) \neq \delta V(-y) \Longrightarrow A_{i\uparrow,j\downarrow} \neq 0$$

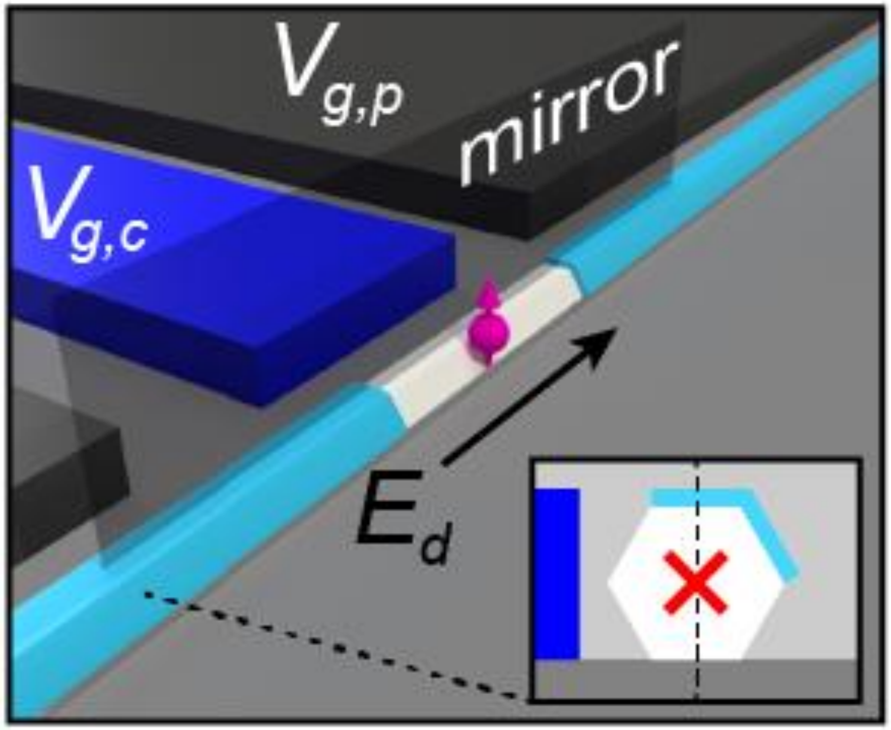


Flux drive

$$A_{i\sigma,j\sigma'} \propto \langle \Phi_{i\sigma}|H_0'|\Phi_{j\sigma'}\rangle \propto \delta_{\sigma,\sigma'}$$

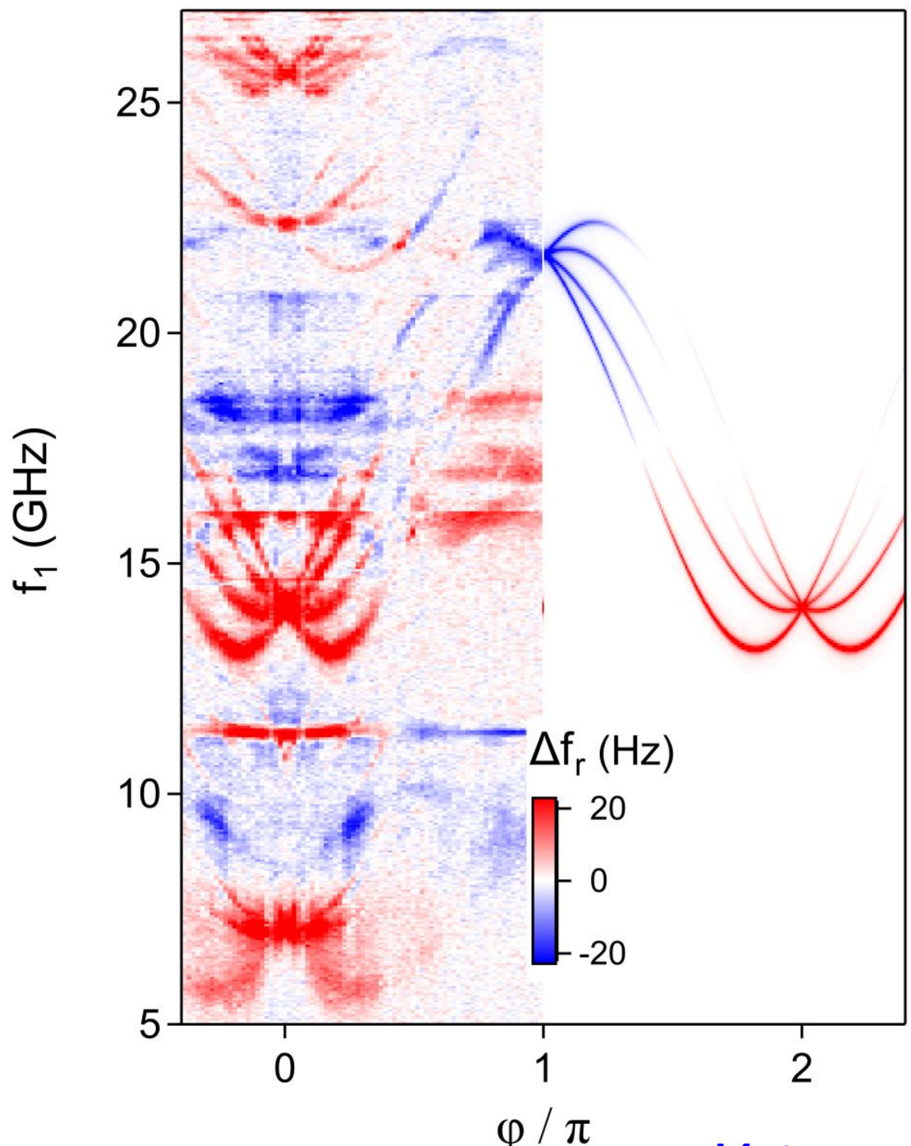
Metzger et al., PRR (2021)

Mirror symmetry breaking by nearby gates and partial Al shells



Hays et al., Science (2021)

Theory of cQED: fit of line intensities for a SQPT



Data from Tosi et al. PRX (2019)

- **Adiabatic regime**

$$f_R \simeq 3\text{GHz} \ll f_1$$

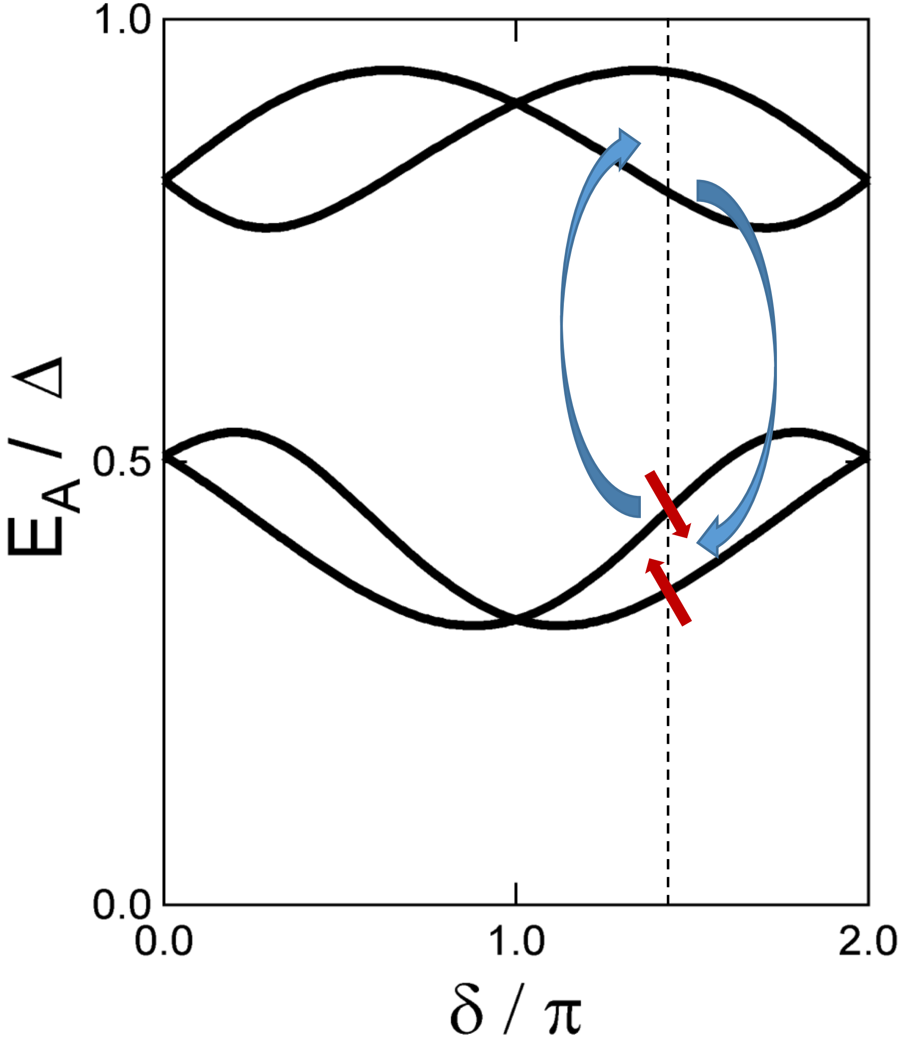
- **No selection rules**

Metzger et al., PRR 2021

The Andreev Spin Qubit (ASQ)

Chtchelkatchev, Nazarov (2003)
Padurariu, Nazarov (2010)
Park, ALY (2017)

Coherent manipulation of an ASQ



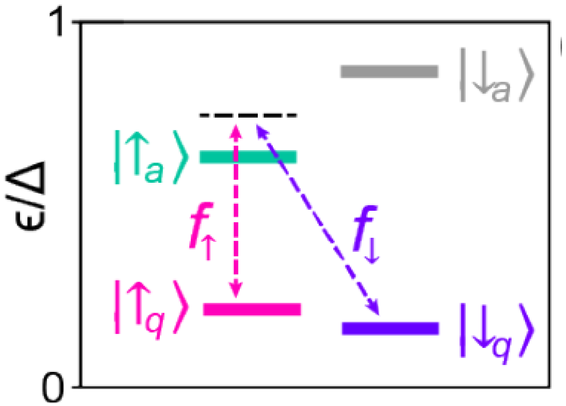
- Direct transitions strongly suppressed

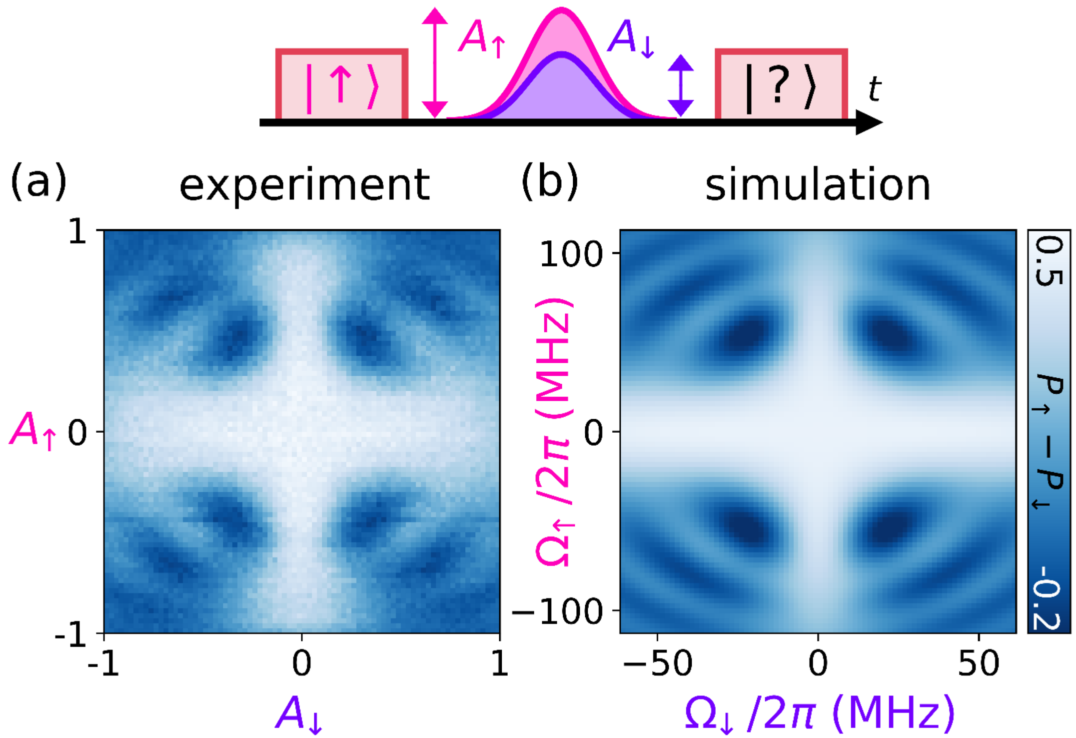
- Idea: Raman transition through higher ABS manifold

J.Cerrillo, M. Hays, V. Fatemi and ALY, PRR (2021)

STIRAP: Stimulated Raman Adiabatic Passage

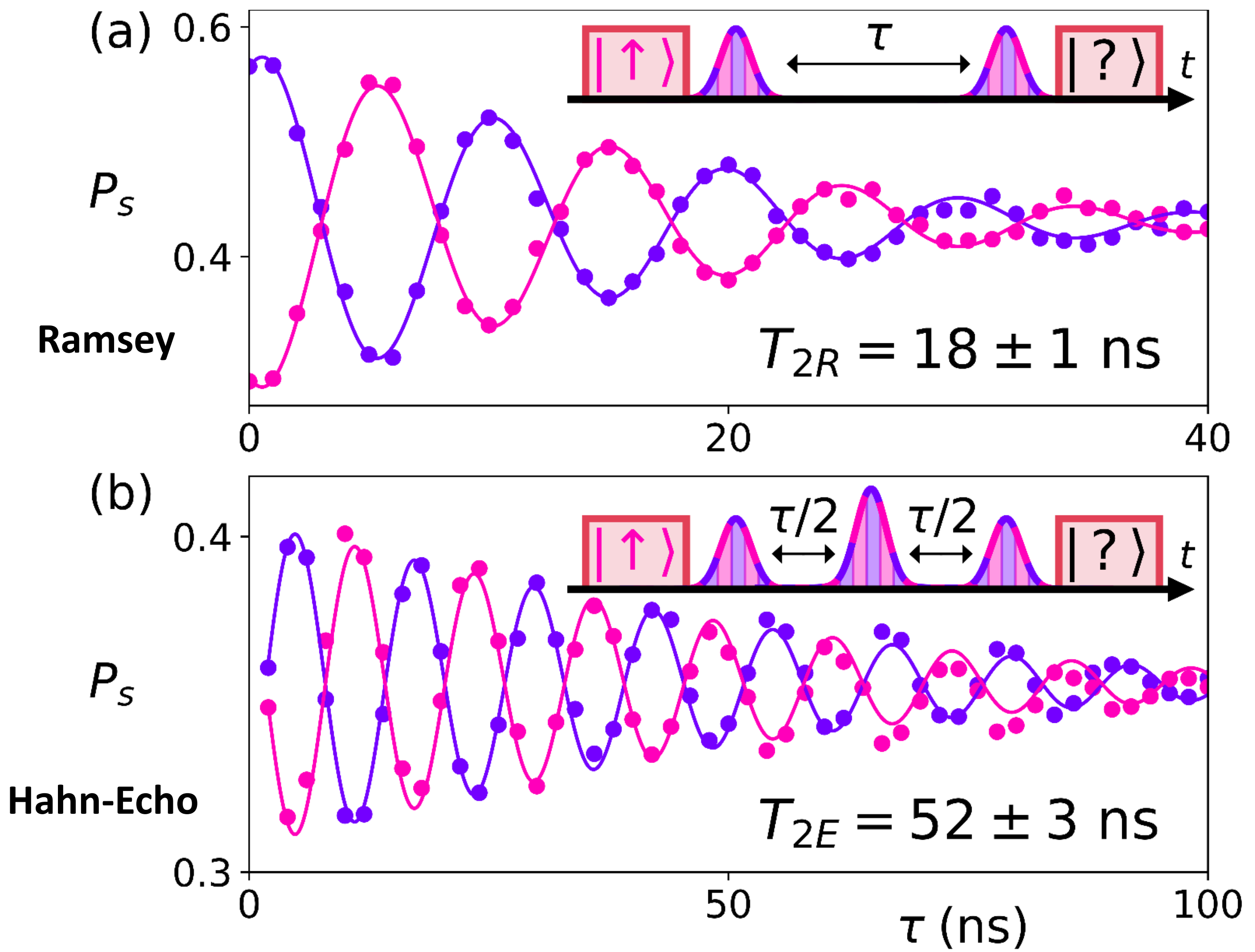
Raman based coherent manipulation of the ASQ





M. Hays, V. Fatemi, D. Bouman, J. Cerrillo, S. Diamond, K. Serniak, T. Connolly, P. Krogstrup, J. Nygård, ALY, A. Geresdi, M. H. Devoret, Science (2021)

Coherence times of the ASQ



M. Hays, V. Fatemi, D. Bouman, J. Cerrillo, S. Diamond, K. Serniak, T. Connolly, P. Krogstrup, J. Nygård, ALY, A. Geresdi, M. H. Devoret, Science (2021)

Conclusions and outlook

- Evolution from transport to cQED techniques in mesoscopic superconductivity
- Atomic contacts: ideal test system for coherent MAR theory
- Extension to TS case: analytical results for NTS, TSTS, STS, etc
- Evidence of “fine structure” of Andreev levels from microwave spectroscopy
- Evidence of (weak) interactions from mixed pair transitions
- Theory of cQED detection. Understanding of line intensities
- Coherent manipulation of trapped quasiparticles pseudospin by means of Raman transitions (Andreev spin qubit)

Conclusions and outlook

Open issues:

- **Origin of excess quasiparticles?**
- **Decoherence mechanisms for the ASQ?**
- **Similar experiments in the topological regime**

Outlook:

- **Multiterminal Josephson-Andreev qubits**

F.J. Matute-Cañadas, L. Tosi and ALY
PRX Quantum (to be published)
arXiv:2312.17305v1

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